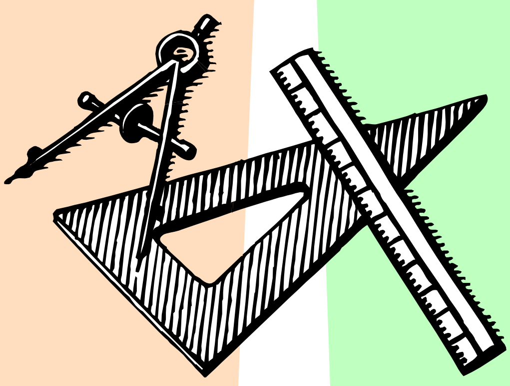
Teaching Measurement Concepts

Grades K–3



2006

Sponsored by: Alberta Education, Learning and Teaching Resources Branch

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Questions or concerns regarding this document can be addressed to Debbie Duvall, Learning and Teaching Resources Branch, Alberta Education. Telephone 780–427–2984. To be connected toll free inside Alberta, dial 310–0000.

Several Web sites are listed in this document. These sites are listed as a service only to identify potentially useful ideas for teaching and learning. Alberta Education is not responsible for maintaining these external sites, nor does the listing of these sites constitute or imply endorsement of their content. The responsibility to evaluate these sites rests with the user.
Note: All Web site addresses were confirmed as accurate at the time of publication but are subject to change.

Acknowledgements

Alberta Education, in partnership with the Alberta Regional Professional Development Consortia, developed a series of workshops for elementary teachers of mathematics. These workshops are designed to provide teachers with strategies for teaching concepts included in the revised Alberta Program of Studies for Mathematics, 2006.

This workshop is one of a series of ten workshops designed for facilitators who will be working with Alberta teachers across the province. Alberta Education would like to acknowledge the contribution of the following individuals:

Dr. Andrea McDonough, Australian Catholic University, writer
Karen Egge, Northern Regional Learning Consortium
Debbie Duvall, Learning and Teaching Resources Branch, Alberta Education
Greg Bishop, Assistant Director, Learning and Teaching Resources Branch, Alberta Education

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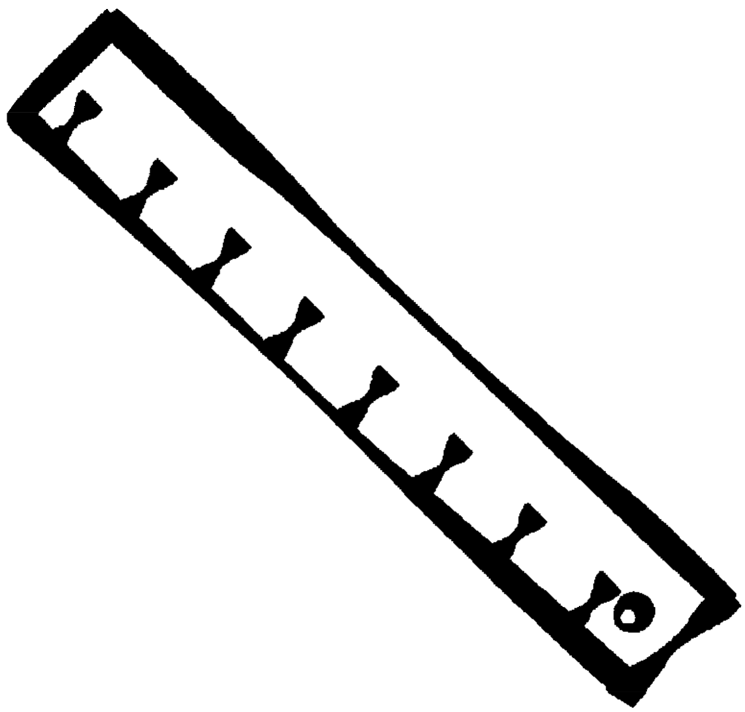

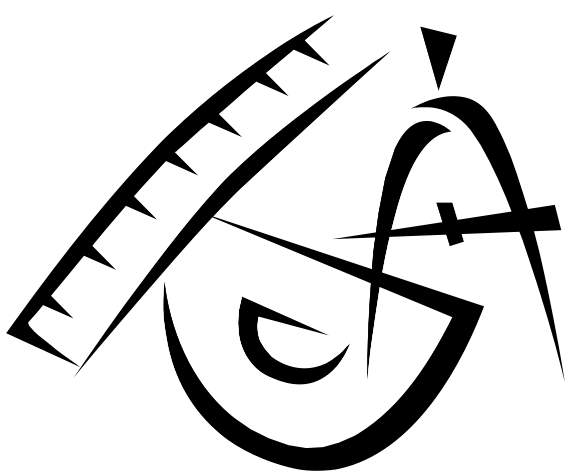

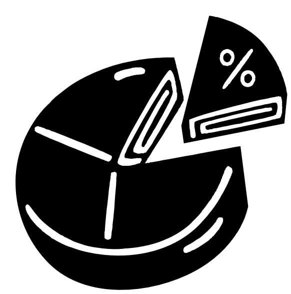

Teaching Measurement Concepts Grades K–3

Workshop Overview

The day consists of a range of activities and reflection on these. The purpose is that participants will:

- think about key ideas in measurement and the importance of students coming to understand these
- become more familiar with the Alberta Curriculum
- reflect on how key ideas in measurement and the Alberta Curriculum guidelines can inform teaching decisions.

Andrea McDonough's paper *Helping children develop key ideas in their learning of measurement* is provided as background for the workshop leader and as reading for teachers.

Details given for each activity include: Materials, Grade, Learner Outcomes, Common Curriculum Framework links, and the Activity. Within the activity, the suggested questioning for use with students is given in italics.

For each activity there is a recording sheet for teachers to use during the workshop. This will facilitate reflection on the features and purpose of each activity, as well as some discussion about how an activity might be adapted. This is an important element of each activity. It is during the discussion of each activity, facilitated by completion of these sheets, that the "Teacher workshop discussion/Raising issues" ideas are available for use. Many of these are supported by slides (as indicated). Also, many of these issues are relevant to more than one activity and may come up in discussion more than once. Some extra slides are provided that list some of the big ideas identified in the McDonough article that have not been discussed within the suggested issues. These may be useful.

The day concludes with a reflection by the teachers on key messages about measurement gained from the day. The intention is that they will work in small groups (e.g., four members) to discuss the day and compose something that indicates what they believe have been some key messages about measurement. The letters of the word "Measurement" are provided as sentence or phrase starters. Teachers may use a large sheet of paper and felt pens to record ideas. The group presents their reflections to the course participants, and the presentation may be a little more creative – such as a poem or song! Teachers also have individual sheets in their packs for their own recording.

Sample Agenda

Full day workshop: 8:30 a.m.–3:30 p.m.

★ Not all of the activities in this binder are included in this sample outline. As a facilitator, you may wish to use different activities from this binder for your workshop.

Time	Activity	Common Curriculum Framework Links		
8:30	Introductions	Outcome	AI	Math Process
8:40	A Bag of Apples	K.1; 1.1	1.1a, 1.1c, 3.3b, 3.3d	C, R, V, PS, CN, ME
9:20	Introduction to the purpose of the day			
9:25	Woolly Worms	K.1; 1.1	1.1a, 1.1b, 1.1c	C, R, V, ME
10:15	MORNING BREAK			
10:20	Make a Teddy Ruler	2.3; 2.4	2.3a, 2.4a, 2.4b	C, R, V, CN, ME
10:50	Teddy Containers	1.1 (2.3)	1.1a, 1.1b 2.3a for capacity	C, R, V, PS, CN
11:30	Calendar Challenge	2.1	2.1b, 2.1d, 2.1g, 2.1h	C, CN
11:50	Guess and Check	3.1, 3.2	3.1a, 3.1b	C, R, V, ME
12:10	LUNCH			

AI – Achievement Indicator

Time	Activity	Common Curriculum Framework Links		
1:10	Mass Activities	2.3, 3.4	2.3a, 2.3b, 3.4b, 3.4e, 3.4f, 3.4h, 3.4i	C, R, V, PS, CN, ME
2:00	Mouse House	3.5	3.5a, 3.5b, 3.5c, 3.5d	C, R, V, PS, CN, ME
2:40	Conclude the day – key messages from the day. (Requires one large sheet of paper per three to four teachers and felt pens)			

AI – Achievement Indicator

A Bag of Apples

Trainer materials:

	Slides
List of questions from another group – use only if necessary	3

Teacher workshop discussion/Raising issues:

	Slides
1. Peeling the apples While it is desirable for students to be accurate when measuring, in this activity you may find that students line up the apple and the string in different ways. (See slides 5 and 6.) The focus here is on being as accurate as possible when comparing the apple peel to the string and when comparing the string to other objects, but with the focus coming to the transitive reasoning. It is worthwhile to discuss with students how we might try to be accurate.	**5 and 6**
2. Transitive reasoning When students are required to make indirect comparisons, they should be encouraged to use transitive reasoning; e.g., if A=B, and B=C, then A=C. This reasoning is used when we compare two things indirectly by using a third object. Teacher questioning makes this reasoning apparent. In this way students can compare, with meaning, two objects (or attributes of one object) that cannot be compared directly. Some other tasks requiring transitive reasoning: – What in this room is the same length as the distance around your head? – Which is longer, the length of your arm or the distance around your head? – Which is longer, the height of your drink bottle or the distance around your head?	**7** **8–11**

Woolly Worms

Teacher workshop discussion/Raising issues:

	Slides
1. Making comparisons	
This activity helps students focus on techniques for making comparisons and on justification strategies. For example, if worms are physically compared, are they straightened and laid beside each other? It is not essential for ends to be lined up, although this can make comparisons easier. This activity gives the opportunity for listening for the correct use of the language of length.	**14**
This slide shows a group of “woolly worms” that cannot as yet be easily compared. Straightening will help.	
As Wilson and Osborne (1992) state, “the method of comparing properties varies from measurement system to measurement system” (p. 92). For example, in length, comparisons may be made by direct covering, or one measurement might be seen to be contained within another (e.g., segment AB is contained by segment AC). However, area	**15–16**
is more complex. While direct comparison is sometimes possible, region R may be longer than region Q, but region Q may be wider than region R, and therefore comparison is more difficult.	**17**
2. Congruence	
This is another key idea in measurement.	**18**
Where two lengths cover each other, with ends corresponding exactly, they can be said to be congruent. Congruence refers to geometric figures and therefore applies to length and area (Wilson & Osborne, 1992).	
3. Conservation	
Conservation refers to the understanding that an object retains its size when moved or subdivided.	**19**
Students in your class will not necessarily conserve length. Some students will think that the more curly worm is shorter that the others when just held up, but longer when straightened out! They may focus on the ends of the worms only. Hands-on experiences such as woolly worms can help students in their development of this concept.	

Make a Teddy Ruler

Teacher workshop discussion/Raising issues:

	Slides
1. Using rulers with understanding *What difficulty have you observed students having with using rulers?* *Are you always convinced that they really understand how a ruler works and what it tells them?* Many students do not use a ruler with meaning. For example, they may read the numbers on a ruler but not understand that the numbers refer to the length of the unit rather than to the line. This activity can focus them on the *unit* and the *iteration* (repetition) of the unit. Students do not always understand this, as demonstrated when some students in the first years of school were asked to construct a ruler (like the 30 cm ruler they had just looked at) using a strip of paper and a pen. The range of responses included writing numbers as far as they could count or with as many as would fit on the strip of card, having only numbers, and not aligning numbers and marks. This task gives students the opportunity to think about measuring involving subdividing a length and iterating, or repeatedly translating, this subdivision. Further tasks, e.g., Making a Footstrip (see extensions), can take this further.	**20** **21–24**
2. Unit, Iteration and Number One key idea in measuring is that of unit. Students need to come to understand that identical units are used to measure. A unit is created by subdividing the whole into congruent parts. The unit is iterated, that is, the unit is translated successively. The count gives the number of units.	**25**
3. There is an inverse relationship between size of unit and number of units Extension Activity 3 can help students see that when measuring an object, the larger the unit, the fewer that are needed, and vice versa. In this case larger refers to longer, but, for example, where mass is measured, larger refers to heavier.	**26**

	Slides
4. Using children's literature (as in Extension Activity 3) Children's literature provides a vehicle to provide meaningful contexts for young students to think about and discuss mathematical ideas. There are many more common children's books that offer potential for use in teaching mathematics. As recommended by Lehrer et al. (2003), the children explore length in a familiar context, in this case provided by the story. The lesson goes beyond the basic idea of the story to develop a tool for measuring length. See, for example, Shih and Giorgis (2004) for a discussion of three different types of literature in which mathematical concepts are successfully integrated into stories. Reference: Shih, J. C., & Giorgis, C. (2004). Building the mathematics and literature connection through children's responses. *Teaching Children Mathematics, 11*(6), 328–333.	

Teddy Containers

Teacher workshop discussion/Raising issues:

	Slides
1. Conservation of capacity The activity is designed to help students see that there is no direct relationship between the capacity of a container and its height alone, i.e., cylinders of different widths and heights can hold the same number of teddies (conservation of capacity). In the lesson share time, containers are laid out on the floor and students make comparisons, finding that containers can be made that look different but have the same capacity. In this way they focus on the concept of conservation of capacity. Following this activity, a student in Grade 2 wrote that, “The first cylinder I made was tall and skinny. The second cylinder was tall and wide. Both of the [sic] had space for ten,” clearly focusing on the concept of conservation of capacity. A similar statement might be made in a class journal, helping each student focus on the key measurement idea in the activity. The teacher plays an important role in identifying the mathematical concept, developing a suitable activity that has this focus, and, through questioning, helping students draw out this big idea of measurement.	**28–29** **30**
2. Having a clear mathematical focus Discuss the importance of having a clear mathematical focus and helping students draw out the key understanding(s) from an activity. Refer to the listing of mathematical activity made with the group part way through the activity (or the list provided on slide 31). It is through students’ identification of the key ideas that we help draw them out and pay attention to the purpose of the lesson.	**31 – use only if necessary**
3. Use of mathematical language Use of terms such as “bigger” provide the opportunity to talk about mathematical language – and help students focus on the concept under discussion and use the correct terminology.	

	Slides
4. Helping students draw out the key mathematical idea from the lesson Case studies of highly effective teachers of mathematics in the first three years of school have shown that one characteristic is drawing out key mathematical ideas during and/or toward the end of the lesson (e.g., McDonough & Clarke, 2003). This might be done, for example, through a class journal or recording by individual students. Reference: McDonough, A., & Clarke, D. (2003). Describing the practice of effective teachers of mathematics in the early years. *Proceedings of the 27th conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 261-268). Honolulu, HI: College of Education, University of Hawaii.	
5. A question that might arise in the teacher discussion: Is it volume or capacity? Discuss the meaning of capacity and standard units for its measurement. Discuss the meaning of volume and standard units for its measurement. In this task, we are measuring and paying attention to how much the containers hold, therefore the focus is the capacity of the containers. Capacity and volume are closely related, as discussed below, but there is also a difference between them. One can think of capacity as “the amount a container holds” (http://mathcentral.uregina.ca/RR/glossary/middle/ Accessed September 6, 2004) or as “the amount a container can hold” (http://www.amathsdictionaryforkids.com/Accessed September 6, 2004). The metric system uses the units millilitre, litre, and kilolitre to measure liquid capacity. A clear definition for volume, and one written for children, is “the amount of space occupied by an object” (http://www.amathsdictionaryforkids.com Accessed September 6, 2004). Reference to the units also helps us see to what we are paying attention. The Macquarie Dictionary describes volume as “the size, measure or amount of anything in three dimensions, the space occupied by a body or substance in cubic units; the SI unit of volume is the cubic metre (m^3)” (Delbridge et al., 2001, p. 105).	**32–33**

	Slides
The difference between volume and capacity is explained on the University of Georgia Web site through reference to examples and nonexamples: > Volume is sometimes used to mean capacity. To distinguish between the two terms, consider two boxes, one that is open and fillable and one that is solid. Let's say the first one is a juice box and the other one is a brick. Liquid can be poured into the juice box, but cannot be poured into the brick because it is solid. So we can say that we can measure the capacity of the juice box, but not the capacity of the brick. Yet we can measure the volume of each the box and the brick. (http://www.intermath-uga.gatech.edu/dictnary/descript.asp?termID=60. Accessed September 6, 2004) Reference: Delbridge, A., Bernard, J. R. L., Blair, D., Butler, S., Peters, P., & Yallop, C. (Eds.). (2001). *The Macquarie dictionary: Federation edition*. Sydney: The Macquarie University.	
6. The features of the task • it is open • it incorporates the use of a range of processes (estimating, measuring, reasoning and strategies and possibly visualizing) • it has a specific mathematical purpose • it has a problem solving element.	

Calendar Challenge

Teacher workshop discussion/Raising issues:

<table>
<tr><th></th><th>Slides</th></tr>
<tr><td>1. What strategies do you use to bring students' attention to the order of
– the days of the week?
– the months of the year?

To learn the order of the days of the week or months of the year, it is important that students have opportunities to recite them in order. When students are beyond Grade 2, it is often assumed that they have this knowledge and it may no longer be emphasized by the teacher. It is something to which students' attention needs to be brought.</td><td></td></tr>
<tr><td>2. Going beyond knowing the order
A question within the Early Numeracy Project Interview showed that while students may know the names of the months in order, they do not necessarily know how one month relates to the next. That is, while they may know the name of the last day in June, and know that July is the next month, they may not be able to say the name of the first day in July when looking at the calendar for the month of June.

It is important that students have access to calendars and use these to solve problems. It is also helpful for students to be able to recall the order of the days and the months when the sequence is broken, e.g., by asking "What is the month after April?"</td><td></td></tr>
<tr><td>3. Duration of time
It is important that students are challenged to think about duration, e.g., by creating timelines. One useful question is: "What are your three favourite times of day? Show them on a timeline that starts when you get up and ends when you go to sleep."

Reference: Sullivan, P., & Lilburn, P. (2004). Open-ended maths activities: Using 'good' questions to enhance learning in mathematics (2nd ed). South Melbourne: Oxford. p. 69.</td><td>34</td></tr>
</table>

Guess and Check

Teacher workshop discussion/Raising issues:

	Slides
1. Duration of time *Can you think of instances where time seems to pass much more quickly than you expect?* For children and adults, actual and perceived time may be different. Perception of the duration of time can be dependant on what you are doing!	
2. The attribute of time for children in K–3 Time is a challenging concept for young children as it is less concrete than other measurement concepts. However, by bringing students' attention to time concepts, even in the early years, we help develop their understandings.	
3. The attribute of time in the curriculum *How do students' experiences during their first years of school fit with what we have done here? How does what we have done link to the K–3 curriculum?* *What other time concepts are identified here?* What are students moving toward? In the Common Curriculum Framework, reading and writing of times is first mentioned in Grade 4. However, students see clocks prior to this. It is important that students have exposure to clocks at earlier levels (although they are not expected to be proficient in reading clocks as yet). Understanding of clocks may help with other outcomes, such as understanding the minute as a unit. It is good to have clocks in the classroom and to refer to them frequently, e.g., "I am going to give you ten minutes to do this activity. The big hand is now on the four. What number will it be on in ten minutes' time? Yes, stop when it reaches the six."	

Mass Activities

Teacher workshop discussion/Raising issues:

	Slides
1. The importance of hands-on activity For students to develop good understandings of measurement concepts, they need to have hands-on activity. It is not enough for them to watch the teacher or to read of measurement scenarios. Indeed, they need to have frequent measurement experiences (Wilson & Osborne, 1992) and reflect on these experiences with teacher support, drawing out the key ideas.	**35**
2. Using everyday materials Using everyday materials and packages, such as a kilogram of sugar, gives students the opportunity to develop referents for common units of measure. Weighing fruits and vegetables also creates interest for the children.	
3. Lesson reflection When students work in rotating groups, it is important that the activity is reflected upon as in any other lesson. One way to facilitate this is to have each of the problems related to the same measurement attribute, as in this case all problems relate to mass. Students can then all contribute meaningfully to the sharing, and make sense of the teacher-led lesson summary. Ideally having a more specific common focus across the activities, such as using nonstandard units or developing personal benchmarks, will help focus the students even more.	

Mouse House

Teacher workshop discussion/Raising issues:

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1. Arrays

While the focus of this activity is perimeter, it is possible that the attribute of area will also arise in the discussion.

The structuring of arrays is commonly seen as a key idea for the development of the area concept (e.g., Stephan & Clements, 2003). Lehrer et al. (2003) state that area key ideas parallel the eight they have identified for length, but note that one of the additional challenges of area measure is the conception of a two-dimensional array composed of units of area measure, typically squares (p. 109).

They note that many students learn to simply multiply products without understanding that these products are generating arrays. This activity can give experience in structuring arrays.

Lehrer et al. (2003) note also that students prefer to treat object boundaries as absolute, believing in principle that units of measure cannot overlap the boundaries. In Extension 2, talking to students as they measure how many floor tiles cover a Mouse House floor of 2 m in perimeter can help address this, especially where they have to take account of overlapping a boundary.

References:
Stephan, M. & Clements, D. H. (2003). Linear and area measurement in prekindergarten to grade 2, In D. H. Clements & G. Bright (Eds.), *Learning and teaching measurement* (2003 Yearbook, pp. 3-16). Reston, VA: National Council of Teachers of Mathematics.
Lehrer, R, Jaslow, L., & Curtis, C. (2003). Developing an understanding of measurement in the elementary grades, in D. H. Clements & G. Bright (Eds.), *Learning and teaching measurement* (pp. 100-121). Reston, VA: National Council of Teachers of Mathematics.

</td><td></td></tr>
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Slides

Teaching Measurement Concepts in Division 1 (K–3) 1	**The Purpose of the Day** That participants will: • think about key ideas in measurement and the importance of children coming to understand these • become more familiar with the Alberta Curriculum • reflect on how key ideas in measurement and the Alberta Curriculum guidelines can inform teaching decisions. 2
Curriculum Outcomes **A Bag of Apples** 1.1: Demonstrate an understanding of measurement as a process of comparing by: • identifying attributes that can be compared • ordering objects • making statements of comparison • filling, covering or matching. [C, CN, PS, R, V] 3	**Curriculum Outcomes** **A Bag of Apples** (continued) 3.3: Demonstrate understanding of measuring length (cm and m) by: • measuring and recording length, width and height. [C, CN, ME, PS, R, V] 4
A Bag of Apples (brainstorm from one group of teachers) • How many apples are there in the bag? • How far will the apples stretch if we lay them in a line? • How long is the peel of one apple? • How long does it take to peel an apple? • How long does it take to eat an apple? • If we laid all the apple peels in the corridor, how far would they go? • How heavy is the bag of apples? • Is every apple the same weight? 5	 6

Transitive Reasoning

The concept of transitivity is necessary when comparing two items where direct comparison is not possible. Use of a third item allows comparison, and is dependent on understanding the following relationships:

If Length A = Length B, and Length B = Length C, then Length A = Length C.

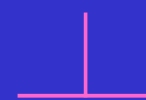
If Length A > Length B, and Length B > Length C, then Length A > Length C.

If Length A < Length B, and Length B < Length C, then Length A < Length C.

7

Helping Children Develop Transitive Reasoning

Which is longer?



8

- Which is longer: The length of your arm or the distance around your head?

9

- Which is longer: The height of your drink bottle or the distance around your head?

10

- Find something in the room that has the same distance around as the distance around your head.

11

Curriculum Outcomes
Woolly Worms

K.1: Use direct comparison to compare two objects based on a single attribute, such as length (height), mass (weight), and volume (capacity). [C, CN, PS, R, V]

1.1: Demonstrate an understanding of measurement as a process of comparing by:
- identifying attributes that can be compared
- ordering objects
- making statements of comparison
- filling, covering or matching.

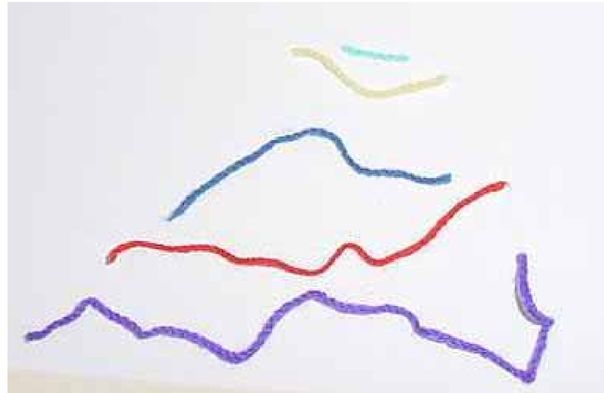
[C, CN, PS, R, V]

12

Woolly Worms

Working in pairs, each make a bag of five woolly worms that are all different in length. Be ready to tell me how you know they are different in length.

13



14

Some Related Tasks

- Compare your longest worm with that of another child. What do you notice? Are your longest worms the same? Why not? Are they both still long worms?
- Put your second longest woolly worm on the table. Your feet are glued to the floor, your bottom is glued to the seat, but we want to find out who has the longest woolly worm of those on the tables! How might we do this?

15

Comparison

- Like properties can be compared to see which is greater.
- "The method of comparing properties varies from measurement system to measurement system."

(Wilson & Osborne, 1992, p. 92)

16

Comparisons

- In length, comparisons may be made by direct covering, or one measurement might be seen to be contained within another (e.g., segment AB is contained by segment AC, therefore AB is shorter than AC).

17

Comparisons

- In area, comparison is more complex.
- Direct comparison is sometimes possible, but not always. For example, region R may be longer than region Q, but region Q may be wider than region R, and therefore comparison is more difficult.

18

Congruence

- Figures are congruent if they coincide when superimposed.
- For example, if one Woolly Worm can be picked up and put down on another so that the endpoints match exactly, they are congruent (both of the same length).
- It is reasonable to think about congruence in relation to length and area. It does not apply to systems such as temperature or time as it applies only to geometric figures (Wilson & Osborne, 1992).

19

Conservation

- An object retains its size when moved or subdivided.

20

Curriculum Outcomes
Make a Teddy Ruler

2.3: Compare and order objects by length, height, distance around and mass (weight) using non-standard units, and make statements of comparison. [C, CN, ME, R, V]

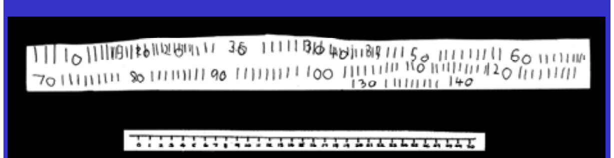
2.4: Measure length to the nearest nonstandard unit by:
- using multiple copies of a unit
- using a single copy of a unit (iteration process).
  [C, ME, R, V]

21

Using Rulers with Understanding

- What difficulty have you observed children having when using rulers?
- Are you always convinced that they really understand how a ruler works and what it tells them?

22



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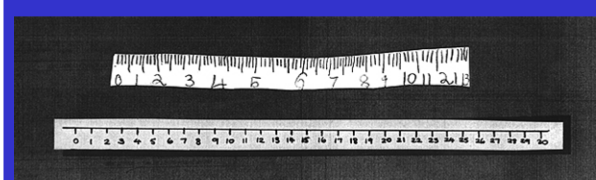

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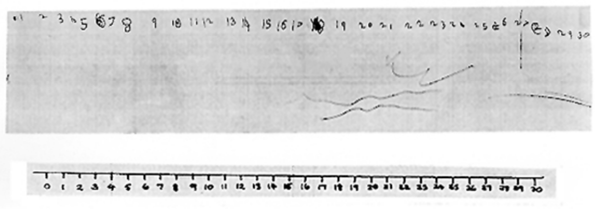

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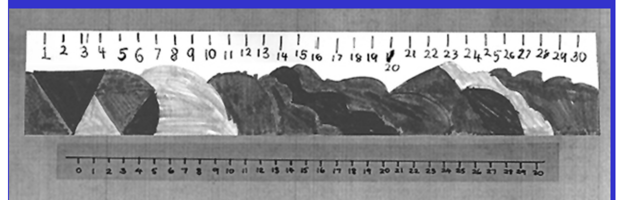


26

Key ideas: Unit, Iteration and Number

One key idea in measuring is that of unit. Children need to come to understand that identical units are used to measure.

A unit is created by subdividing the whole into congruent parts. The unit is iterated, that is, the unit is translated successively.

The count gives the number of units.

27

c

Size of Unit Compared to Number of Units

- There is an inverse relationship between the size of a unit and the number of units.

28

Curriculum Outcomes
Teddy Containers

1.1: Demonstrate an understanding of measurement as a process of comparing by:
- identifying attributes that can be compared
- ordering objects
- making statements of comparison
- filling, covering or matching.

[C, CN, PS, R, V]

[Could also apply the following objective, but to Capacity:

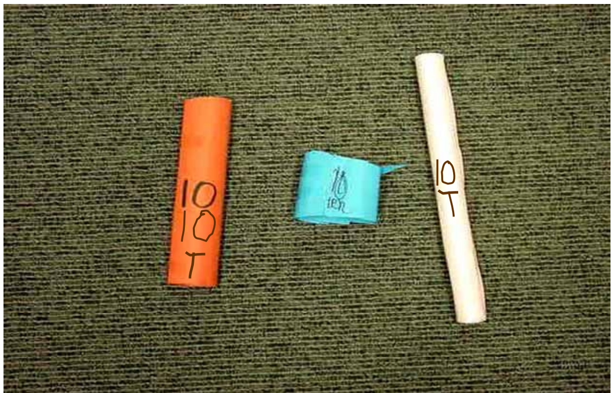
2.3: Compare and order objects by length, height, distance around and mass (weight) using non-standard units, and make statements of comparison. [C, CN, R, V]

29

Teddy Containers

Make a cylinder that holds exactly ten teddies

30



31

The children placed their teddy cylinders on the floor and described what they observed.

32

A Grade 2 Child Wrote:

"The first cylinder I made was tall and skinny. The second cylinder was tall and wide. Both of the [*sic*] had space for ten."

33

The Mathematics in the First Half of the Activity

(A brainstorm from one group of teachers)

- Estimating
- Counting
- Part whole (colours in the container)
- Shapes – e.g., cylinder
  - naming
  - making by rolling
- Packing
- Sorting (by colour)
- Approximating
- Checking
- Multiplication
- Measuring
- Measurement principles
- Comparing

34

Is it Volume or Capacity?

- Capacity: the amount a container holds.
- Volume: the amount of space occupied by an object.

35

- Consider two boxes, one that is open and fillable and one that is solid, e.g., a juice box and a brick.
- Liquid can be poured into the juice box, but liquid cannot be poured into the brick because it is solid.
- We can measure the capacity of the juice box, but not the capacity of the brick. Yet we can measure the volume of each the box and the brick.
- http://www.intermath-uga.gatech.edu/dictnary/ descript.asp?termID=60

36

Curriculum Outcomes
Calendar Challenge

2.1: Relate the number of days to a week and the number of months to a year in a problem solving context.
[C, CN, PS, R]

37

Duration of Time

"What are your three favourite times of day? Show them on a timeline that starts when you get up and ends when you go to sleep."

(Sullivan & Lilburn, 2004, p. 69)

38

Curriculum Outcomes
Guess and Check

3.1: Relate the passage of time to common activities using non-standard and standard units (minutes, hours, days, months and years). [CN, ME, R]

3.2: Relate the number of seconds to a minute, the number of minutes to an hour and the number of days to a month in a problem solving context.
[C, CN, PS, R, V]

39

Curriculum Outcomes
Mass Activities

2.3: Compare and order objects by length, height, distance around, mass (weight) using non-standard units, and make statements of comparison.
[C, CN, ME, R, V]

3.4: Demonstrate an understanding of measuring mass (g and kg) by:
- selecting and justifying referents for the units g and kg
- modelling and describing the relationship between the units g and kg
- estimating mass using referents
- measuring and recording mass.

[C, CN, ME, PS, R, V]

40

Wilson & Osborne (1992) recommended:

- that children should measure often, preferably on real problems
- that children should have activity-oriented encounters with measurement rather than observing
- that instruction should emphasize the important ideas of measurement that transfer across measurement systems.

41

Curriculum Outcomes
Mouse House

3.5: Demonstrate an understanding of perimeter of regular and irregular shapes by:
- estimating and recording perimeter (cm and m)
- constructing different shapes for a given perimeter (cm, m) to demonstrate that many shapes are possible for a perimeter.

[C, ME, PS, R, V]

42

M……………………………………..
E……………………………………..
A……………………………………..
S……………………………………..
U……………………………………..
R……………………………………..
E……………………………………..
M……………………………………..
E……………………………………..
N……………………………………..
T……………………………………..

43

The End

44

Additivity

- "Measurement of parts can be added to obtain the measurement of the whole"
  (Wilson & Osborne,1992, p. 94).

A line segment can be divided into smaller line segments whose sum equals the original length.
(Lehrer, 2003).

45

Accumulation of Distance

- "The result of iterating a unit signifies, for students, the distance from the beginning of the first iteration to the end of the last"
  (Stephan & Clements, 2003, p. 6).

46

Origin/Zero Point

- When using a scale to measure, it is important to identify the zero point, and that any point can serve as the zero point or origin on a scale.
- For example, the distance between 0 and 10 is the same as the distance between 30 and 40.

47

Partition/ing

- understanding that units can be partitioned
- Stephan and Clements (2003) describe partitioning as "the mental activity of slicing up the length of an object into the same-size units" (p. 4).

48

Precision

- A key idea for children to come to understand is that the choice of unit determines the level of precision.

49

Proportionality

- This understanding is that different-sized units can be used to measure the same thing. Thus different quantities can represent the same measure, these quantities being inversely proportional to the size of the units used.

50

Relation to Number

- Stephan and Clements (2003) state that an important understanding is that "Measurement is related to number in that measuring is simply a case of counting. However, measurement is conceptually more advanced since students must reorganize their understanding of the very objects they're counting (discrete versus continuous units)" (p. 7).

51

Standardization

- The use of standard units facilitates communication of measures.

52

Tiling

- Tiling refers to the idea that the units must fill the space, gaps must not be left between the units and there should be no overlaps.

53

Unit/Unit-attribute Relations

- The unit needs to be compatible with the property or attribute to be measured.

54

Elementary Mathematics Workshop Feedback Form

Teaching Measurement Concepts: Grades 1–3

What I liked best about the workshop:

____

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____

What I would like to see changed in this workshop:

____

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Other general comments:

____

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____

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Comments on specific lessons:

Name of lesson: ______________________________________

Comments: ____

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Comments on specific lessons:

Name of lesson: ______________________________

Comments: ______________________________

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Comments on specific lessons:

Name of lesson: ______________________________

Comments: ______________________________

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Comments on specific lessons:

Name of lesson: ______________________________

Comments: ______________________________

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Comments on specific lessons:

Name of lesson: ______________________________

Comments: ______________________________

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The last word is yours:

____

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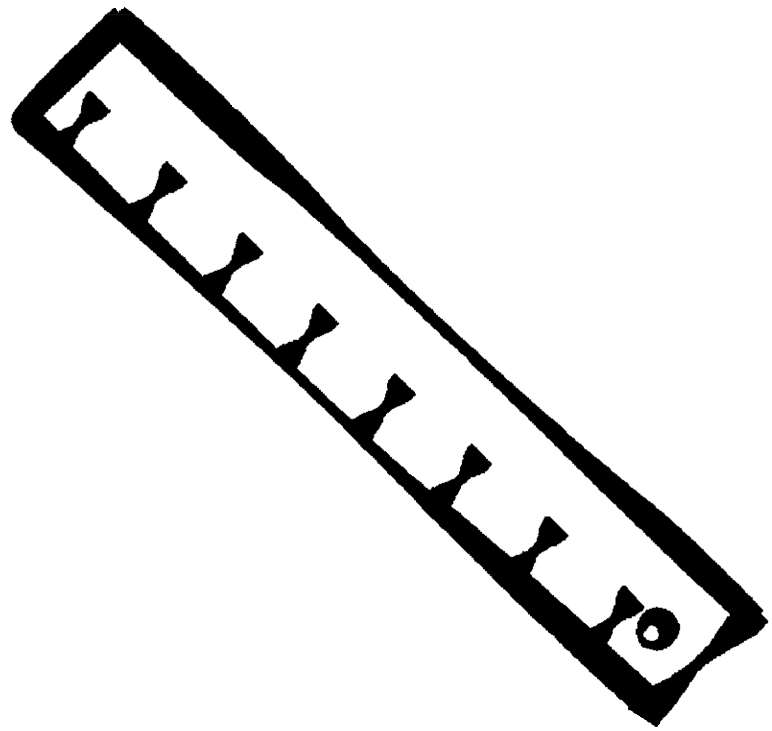
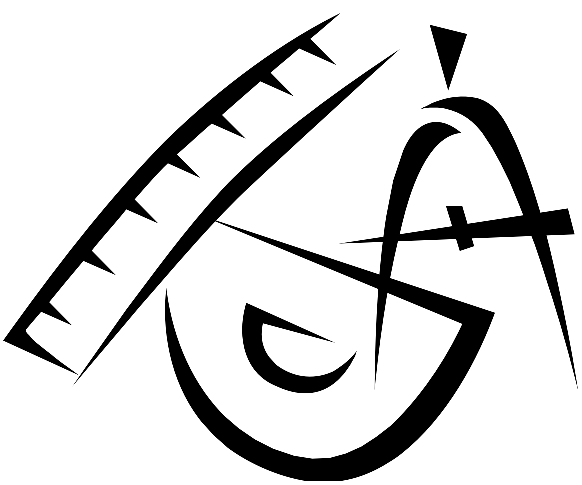
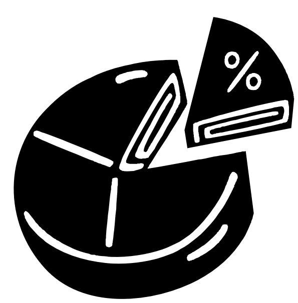
Name: (optional)
Contact information: Telephone: __________ E-mail: ______________________

Activities







A Bag of Apples

Materials:

- Approximately 15 apples (in a clear plastic bag)
- Extra apples, so that there is one per person
- Vegetable peelers (one per person)
- String – enough for about 1.5 m per person
- Scissors
- Paper plates or paper serviettes/napkins
- Metre ruler
- 30 cm rulers – one per child
- Writing materials for students to make a record of what they have learned

Grade: 1, 3

Learner Outcomes:

Grade 1, No. 1

Demonstrate an understanding of measurement as a process of comparing by:

- identifying attributes that can be compared
- ordering objects
- making statements of comparison
- filling, covering or matching.

[C, CN, PS, R, V]

Grade 3, No. 3

Demonstrate understanding of measuring length (cm and m) by:

- measuring and recording length, width and height

[C, CN, ME, PS, R, V]

Activity Description:

Students explore length within an engaging context and through physical involvement. They make an estimate of the length of an apple peel and decide what in the room might be the same length. In this way, estimation involves visualization. Once the peel is produced (by peeling the apple), students use a third item (string) to compare the length of their peel to other things in the room. Through discussion of their findings, comparative reasoning is encouraged. The extension ideas allow for the use of nonstandard and standard units if desired.

The Activity:

1. Warm up

Hold up one bag of apples (approximately 15 in see-through plastic bag).
What are some questions you could ask about the apples?
Record (for later reference)

Do at least one activity from the questions listed, e.g., *how many apples are in the bag?* Have students estimate and tell how they made their estimation. As the apples are taken out of the bag, you might also ask: *Where do you think the row of apples will reach on this table?* One strategy is to have three students put a tag where they think, and others then vote for the one that is closest to their estimate. Students also describe how they made their estimate.

Choose one measurement activity (as below) or introduce this as something you had thought of for today.

2. Introduction to apple peel activity

Tell a story, such as: *My friend had an uncle who loved peeling apples and making the peel into one strip as long as he could! My friend loved to peel her apples too. Sometimes the peel came out in lots of pieces, but as she had more practice she got better at making just one piece!*

We are going to think about peeling apples. Here is an apple and you have an apple. Look closely at your apple and imagine that you will peel your apple. Think about your peel making one strip (it might be in pieces but then imagine they are placed in a line to make a strip). Make a picture in your mind of how you think your peel will look.

Now look around the room. What at the other side of the room do you think will be the same length as your apple peel? (It is important that students choose something that will require them to move from their seats – and not be able to take the apple peel!)

Each student identifies at least one object.

How could we find out whether the apple peel and the thing you chose are the same length?

What if your peel is in pieces? It will be very difficult to carry around. What could you use to help you find something of the same length? (If they suggest a ruler, ask what could be used if you do not have a ruler).

Students each peel one apple (it is okay if the peel is in pieces).

They each cut string to the total length of their apple peel (there may be discussion as to where the string should be placed, e.g., along the centre of the peel or otherwise).

Ask: *Walk around the room and use the string to find an object that is the same length as the apple peel*

3. Discussion

What did you find that is the same length as your apple peel?

How do you know it is the same length (when you could not take the peel around the room)? Please explain to us.

Draw out the idea that if the apple peel is the same length as the string and the string is the same length as, say, the table top, the apple peel and the table top must be the same length, that is, if A=B, and B=C, then A=C. (This reasoning is used when we compare two things indirectly by using a third object.)

How did you choose your object? (Strategies)

4. Using standard units

Ask students: *What do you know about one metre? What in this room do you think is about one metre in length?* (other than the metre ruler)

Do you think your apple peel is more or less than one metre in length?

How long do you estimate your apple peel to be?

Students record their estimate.

How did you decide? What helped you?

How could you check?

Today I would like you to find out the length of your apple peel by using 30-cm rulers. Please work in pairs.

Note whether students iterate the ruler correctly and whether they continue to pay attention to the equivalence of 1 metre and 100 centimetres.

5. Consolidation of learning

Please make a record of all you have learned in this apple peel activity today. You may need to write and draw diagrams.

6. Discussion

What did you find? So what can you say about your apple peel? What can we say as a class about our apple peels and what we did today? What comparisons can we make? How could we decide who has made the longest apple peel? The second longest apple peel? etc.

Possible Alternate Activity:

Materials:
- Nonstandard units to measure length

Grade: 2

Learner Outcomes: Grade 2, No. 3

Compare and order objects by length, height, distance around and mass (weight) using nonstandard units, and make statements of comparison.

[C, CN, ME, R, V]

Activity:

- *How many paperclips (or some other linear concrete material) long is your apple peel? (nonstandard units)*

Relevant Reference:

Cheeseman, J., & Ward, J. (2001). A bag of apples. *Australian Primary Mathematics Classroom, 6*(4), 19–21.

* I am indebted to my former colleague in the Early Numeracy Research Project, Jill Cheeseman, for the idea of using a bag of apples as a context for a mathematics lesson with young children.

Woolly Worms

Materials:

- Whiteboard and markers, to record student's words about worms
- Four sample woolly worms in a sandwich bag (one should be more curly than the others)
- Small balls of wool (or similar material)
- Scissors – one pair for each pair of students
- Ziplock sandwich bag for each student

Optional:

- Real worms (or pictures of)

Grade: K, 1 (extension to Grade 2)

Learner Outcomes:

Kindergarten, No. 1

Use direct comparison to compare two objects based on a single attribute, such as length (height), mass (weight), and volume (capacity).

[C, CN, PS, R, V]

Grade 1, No. 1

Demonstrate an understanding of measurement as a process of comparing by:

- identifying attributes that can be compared
- ordering objects
- making statements of comparison
- filling, covering or matching.

[C, CN, PS, R, V]

Activity Description:

Students again explore length within an engaging context. The making of a set of five woolly worms provides each student with materials to conduct direct and indirect comparisons. The language of length and particularly of comparison is explored within this activity. The idea of *conservation of length* is also developed within this task. The extension gives the opportunity for the use of nonstandard units.

The Activity:

7. Introduction

- *What do you know about worms? Does anyone have any worms at home? Where are they?* (In the garden)
- *How would you describe them?*
- Record students' words.
- Students then act being worms.
- *I don't have any real worms but today I brought some very special worms – woolly worms!*
  - Show a sample sandwich bag that holds three woolly worms (two of the same colour, one of a different colour, and all of different lengths. These are the three straighter woolly worms). Have three students hold up these worms at the front of the group.
- Ask the group: *What can you tell me about these worms? How are they the same? How are they different?*
- Include difference in length within the discussion. Ask the three students to order the three worms from shortest to longest. Discuss strategies for ordering. Introduce a fourth, more curly, woolly worm. Ask: *Where will it go in the order and why?* Encourage students to visualize and reason. Typically the discussion will centre on the importance of straightening the worms, of holding them side by side to compare (and possibly matching ends). A student is chosen to find out where this worm will go to be in correct order.

8. Making Woolly Worms:

- Pose the problem: "*Work in pairs to make a set of five woolly worms that are all different in length. Be ready to tell me how you know they are different in length.*

Using scissors and small balls of wool of different colours, students work in pairs, one holding the wool and the other cutting, but each creating their own set of five woolly worms (to be stored in their own sandwich bag!). It would be good for each student to make their set all in one colour and in a colour that is different than the colour used by other students in the group.

As students work on the problem, move around the room, observing and listening as they create their worms. Ask them to justify how they know their worms are different in length.

9. **Ordering the Worms:**

- Have each pair of students order their worms from longest to shortest.
- When they have finished the task, ask groups to pair up to see if they agree with the ordering of the other groups. If they do not, have the students discuss their reasoning for the order and come to agreement. You may wish to involve the whole class in the discussion.
- Have students share their strategies for ordering their worms.

Extensions:

A Challenging Problem

Pose the question of comparing worms made by two students when the students are sitting at different tables and the worms cannot be transported around the room. Challenge students to decide how they would do this. They might choose nonstandard or standard units. If they decide to use nonstandard units, ask what they might have to think about in using them. If they decide to use a ruler, observe to see if they use a correct technique.

Measuring worms with nonstandard units and standard units

Materials:

- Plain paper
- Adhesive tape
- Nonstandard units (e.g., Unifix)

Grade: 2

Learner Outcomes:

Grade 2, No. 3
Compare and order objects by length, height, distance around and mass (weight) using nonstandard units, and make statements of comparison.
[C, CN, ME, R, V]

Grade 2, No. 4
Measure length to the nearest nonstandard unit by:
– using multiple copies of a unit
– using a single copy of a unit (iteration process).
[C, ME, R, V]

Activity:

How long is your worm?

- *If we wanted to measure the length of one worm, what might we use to measure it?* (Brainstorm ideas such as unifix cube strips, paper clips, etc.)
- Show the students how to measure the length of your own worm using nonstandard units. Ask, *"How do you know I measured all the length of my worm? Where did I start measuring from? How did I place the paper clips to make sure that I measured the entire worm?"*
- Have students tape their worms to a piece of plain paper, then measure their worms with nonstandard units and record their findings. Note whether they follow the principles of measuring. Do they record both the number and the unit?
- As well as asking questions such as *How many paper clips long is this blue woolly worm?*, we might ask questions such as, *"How much longer is the blue woolly worm than the red woolly worm?"* In this way students are encouraged to reason.
- Pose the question of comparing worms made by two students when the students are sitting at different tables and the worms cannot be transported around the room. Challenge students to decide how they would do this.

Only one unit:

Pose the problem to the students: *"I want to measure my worm but I only have one paper clip. How might I find out how many paper clips long my worm is?"*

- Discuss how this could be done.
- Have students work in pairs to measure one of their worms using the iteration process. Discuss the difficulties of doing this.

Related Reference:

McDonough, A., Cheeseman, J., & Clarke, D. (2003). "Woolly worms," in G. W. Bright & D. H. Clements (Eds.), *Classroom activities for teaching and learning measurement* (2003 Yearbook of the National Council of Teachers of Mathematics, pp. 3-6). Reston, VA: NCTM.

How Long Is Your String?

Materials:

- Overhead with question, "How long is your string?"
- Recording sheet for each student (see pg. 41)
- A bag containing several pieces of string of different lengths (and colours) for each group of four students

Note:

- Cut pieces of string in lengths that are equal to the length of common objects in the room, such as the blackboard ledge, the width of a table, the length of a metre stick, the length of a pencil, etc.
- Each group should have strings of about the same lengths. It is better to use heavy string than wool since wool tends to curl and heavy string will not.

Grade: 1, 2

Learner Outcomes:

Grade 1, No. 1

Demonstrate an understanding of measurement as a process of comparing by:

– ordering objects
– making statements of comparison
– matching.

[C, CN, PS, R, V]

Grade 2, No. 3

Compare and order objects by length, height, distance around and mass (weight) using nonstandard units, and make statements of comparison.

[C, CN, ME, R, V]

Grade 2, No. 4

Measure length to the nearest nonstandard unit by:

– using multiple copies of a unit
– using a single copy of a unit (iteration process).

[C, ME, R, V]

Activity Description:

Students explore length within an engaging context and through physical involvement. They make an estimate of the length of a piece of string and decide what in the room might be the same length. In this way, estimation involves visualization. Students then verify their estimation by matching the length of their string to objects in the room. Through discussion of their findings, comparative reasoning is encouraged. Students describe their string using comparative language. The extension ideas allow for the use of nonstandard and standard units if desired.

The Activity:

Instructions:

1. Have students work in groups of three or four. Provide each group with a bag containing lengths of different colours of string. Each student reaches into the bag and pulls out one piece of string.

2. Post the question on the overhead, "*How long is your string?*"
   - Tell the students that you want them to be able to describe how long their string is by comparing it to common objects in the room.
   - Ask students to look around the room to find something in the room that appears to be almost the same length as their string. Students may select more than one object.
   - Once students have selected an object that appears to be the same length of their string, ask, "*How will you check to see if the string really is almost the same length as the object you chose?*" Discuss as a class/group.
   - Have the students compare their string to the objects in the room to find one that is almost the same length. When they have found one, they should return to their group.

3. Which string is mine?
   - Have the students place their strings on the floor beside one another. Students take turns describing their string as, "my string is almost the same length as (name of object)." The other students in the group must guess which string the student is referring to. Continue with other members of the group describing their strings.
   - Have some students share their descriptions with the class. This time, ask, *"Does anyone have a string that is the same length as mine?"* Students in other groups should try to find which string is the same length from the strings in their group.
   - Ask *"How can we check to be sure you are right?"* Discuss how the students might check to be sure the strings are the same length.

4. Mine is longer/shorter than …
   - Back in their groups, have students mentally choose one string by colour and keep this string in their mind. The student then describes the string as being longer than (other colours of string) or shorter than (other colours of string). The other students in the group try to guess which string the student is referring to. Repeat with other students taking turns.

Extension: (Indirect Measurement)

1. Tell students you want to find as many objects as possible that are the same length as their desk. Since it is not possible to take the desk around the room to compare the length of objects by matching, ask the students how you might be able to use their knowledge of comparing string to objects to help with this task.
   - Draw out the ideas that if we cut a piece of string the same length as the length of the desk, we can use the string to find objects of equal length. If the string is the same length as the desk and the same length as the bulletin board, then the desk and the bulletin board will be the same length. That is, if A=B, and B=C, then A=C. This reasoning is used when we compare two things indirectly by using a third object when the relationships are identified.

2. Have students select an object such as their desk top, or another object that cannot easily be moved. Have them use string to help then find objects that are the same length using indirect comparison. Have students record their findings:

> My desk is the same length as: ______________
> ______________
> ______________
>
> *(Draw picture of the objects they found to be the same length)*

Make a Teddy Ruler

Materials:

- Plastic teddies all the same size (one per student)
- Strips of paper – longer than 10 teddies in length
- Scissors
- Pens or pencils

Grade: 2

Learner Outcomes:

Grade 2, No. 3

Compare and order objects by length, height, distance around and mass (weight) using nonstandard units, and make statements of comparison.

[C, CN, ME, R, V]

Grade 2, No. 4

Measure length to the nearest nonstandard unit by:

– using multiple copies of a unit
– using a single copy of a unit (iteration process).

[C, ME, R, V]

Activity Description:

Students are each given one teddy and a strip of paper. They are challenged to find an object in the room that is ten teddies long. To do this, one strategy is to make a teddy ruler by moving the teddy along the strip of paper and marking the top of the teddy each time until they have marked ten teddy lengths. Thus the strip becomes a ruler that the student has created where the unit is the length of a teddy. Notions of unit, iteration, and assigning a number are elements of this activity. The activity is designed to help students make links from using nonstandard to using standard units through creation of a scale, and thus develop an understanding of the ruler as a measuring tool. Teachers have commented that this is a very effective activity and one that students greatly enjoy. They love to measure objects both at home and at school with their newly created teddy ruler.

Reference:

The idea of creating a teddy ruler came from:

Graham, E. (1990). *Mathematics for teddy bears: Problem solving activities for young children*. Essex: Claire Publications.

The Activity:

1. Introduction

- *Today we are going to measure objects in the room with teddies.*
- *What do you think in this room might be ten teddies long? How did you decide that it is ten teddies long?*
- *How could we check? What do we need to think about when we are measuring with teddies?*
- *What if we only have one teddy each? What could we do then?*
- Students suggest strategies and offer ideas on why.
- *Today you have a teddy, a strip of paper and a pencil. How might we find objects that are ten teddies long?*

If students do not come up with the idea of repeating the teddy and marking the paper, offer this as one suggestion.

2. Making the ruler and measuring

- Students each make a teddy ruler using one teddy and iterating the teddy.

Using their personal teddy ruler, they measure objects in the room to find objects that are ten teddies long. During this time, question students as to objects that are less than ten teddies long and more than ten teddies long.

For students who find objects quickly, challenge them to find an object in the room that is 15 teddies long. Once they have done this, ask them to explain how they know the object is 15 teddies long. (This task challenges them to iterate their ruler and use counting on.)

3. Reflecting on their learning

- Students share the object they have found that is ten teddies long.

If differences are found, use this as a learning situation. Pose a relevant question that asks students to reflect on a difference, such as: *Susie found that the teacher's pen box is ten teddies long, but Matthew measured it as nine teddies long. How might this have happened?*

This helps students think about important considerations in making and using a ruler, such as using the same teddies (i.e., using the same size unit), laying the teddies in the same direction (i.e., using the unit in the same way), not leaving extra gaps (i.e., iterating carefully so as not to change the size of the unit), and recording the numbers after each teddy (i.e., each time seeing the whole teddy as the unit).

Extensions

1. Create Other Teddy Rulers

Materials:
- Plastic teddies all the same size but different from those used earlier (one per student)
- Strips of paper – longer than ten teddies in length
- Pens or pencils
- Scissors

Grade: 2

Learner Outcomes: Grade 2, No. 2
Relate the size of a unit of measure to the number of units (limited to nonstandard units) used to measure length and mass (weight).
[C, CN, ME, R, V]

Activity Description: Using a teddy of a different height (length), students create a teddy ruler and comment on the different findings when measuring objects and why.

2. Create a Footstrip

Materials:
- Paper – enough for each student to cut out five shoe tracings
- Adhesive tape

Lehrer et al. (2003) suggest that children start by using walking as a means of measuring a length in the classroom with the teacher asking questions such as *"If we all measured the same wall, why are the number of feet different?"* Individual footstrips, where children draw around their shoes and tape five shoe prints together (McClain et al., 1999) can then be made. This footstrip is used, through iteration, to measure objects of various lengths. Lehrer at al. state that this manufacture of a tool helps children make the transition from movement to measuring.

References relevant to Alternate Activity/Extension 2:

Lehrer, R, Jaslow, L., & Curtis, C. (2003). "Developing an understanding of measurement in the elementary grades," in D. H. Clements & G. Bright (Eds.), *Learning and teaching measurement* (2003 Yearbook, pp. 100–121). Reston, VA: National Council of Teachers of Mathematics.

McClain, K., Cobb, P., Gravemeijer, K., & Estes, B. (1999). "Developing mathematical reasoning within the context of measurement," in L. V. Stiff & F. R. Curcio (Eds.), *Developing mathematical reasoning in grades K–12* (1999 Yearbook, pp. 93–106). Reston, VA: National Council of Teachers of Mathematics.

How Big Is A Foot?

Materials:

- Storybook: *How Big Is A Foot?* By Rolf Myller (ISBN# 0–440–40495-9)
- 2 strips of different coloured paper per student (each approximately 25 cm in length, and approximately 10–15 cm wide)
- Scissors
- Adhesive tape
- One felt pen

Grade: 2

Learner Outcomes:

Grade 2, No. 2
Relate the size of a unit of measure to the number of units (limited to nonstandard units) used to measure length and mass (weight).
[C, CN, ME, R, V]

Grade 2, No. 3
Compare and order objects by length, height, distance around and mass (weight) using nonstandard units, and make statements of comparison.
[C, CN, ME, R, V]

Grade 2, No. 4
Measure length to the nearest nonstandard unit by:
– using multiple copies of a unit
– using a single copy of a unit (iteration process).
[C, ME, R, V]

Activity Description:

Students are challenged to reflect on a measuring situation and are led to the idea of using units of measure. The activity is designed to help students make links from using nonstandard to using standard units for measuring length.

The Activity:

Sharing the Story

- Show the cover of the book *How Big is A Foot*.
- Ask: *What do you think this will be about? What do you think will happen?*
- Discuss what happened and why they think that happened:
  - Pause and discuss the carpenters' question, "How big is a bed?" *How big is a bed? Are all beds all the same size? How could you find out how big your bed was?*
  - Pause again after the king measures the queen using his feet and ask, *"Do you think that was a good way to find out how big the bed needs to be? Why or why not?"*
  - When the queen tries the bed and it is too small, have students discuss how this might have happened. Have them suggest ways to fix the situation.
  - Pause again after the carpenter makes a new bed using the king's sculptured foot. Have students predict if the bed will fit the king and explain their reasoning.

Discussing the Concepts

- After reading the last page, discuss why they decided to use a copy of the king's foot to measure from then on. Ask, *"Do we still use copies of the king's foot to measure length? If not, what do we use?"*
- You may wish to use some of the following questions for further discussion.

  1. *Why was the bed too small?*

  2. The king used his feet to measure the Queen's length in order to pass on the dimensions to the apprentice carpenter. What else could the King have used to measure the Queen?

  3. *The King used his feet to measure the Queen for the bed. Have students trace around their feet or shoes on a piece of paper. Have them cut out the foot and use it to measure various items in the classroom. Be sure to have them estimate the measurements first.*

  4. Have students measure the sizes of various body parts using nonstandard units, e.g., circumference of their heads, waist sizes, wrists and ankles. Measure from head to toe, from fingertip to fingertip.

  5. *The King needed to measure the Queen to get the right size bed. Why do people need to measure the Queen to get the right size bed? Why do people need to use linear measurement?*

Making Foot Lengths

- *Let's try it out! Today we will do the same thing – but let's have both a king and a queen.*

Choose two students to be the King and Queen. (Try to have two students with feet of quite different lengths – one with long feet and one with short feet).

- Using their strips of paper, all the students in the class cut one strip to the length of the King's foot and one strip to the length of the Queen's foot. (It is good if half the strips for the Queen's foot are of one colour and half another colour so that a ruler of alternating colours can be constructed. The same applies for the King's foot strips).
- *Making and measuring with rulers, let's see how many King's feet long the (cupboard) is.* (Lay pieces next to the cupboard.)
- *What might be an easier way to do this?* (Tape together strips of alternating colours.)
- Do the same for the Queen's foot length strips.
- *How many King's feet long do you think Corey will be?* Students estimate.
- Corey lies down on the floor and the ruler is placed next to him.
- *How many Queen's feet long do you think Corey will be? Why do you think this?*
- Measure Corey to find out.
  - *What did we find?*
  - *How tall is Corey?*
  - *Why do you think that when we measured Corey there were different numbers of King's and Queen's feet?* (This brings in the idea that we use less of a larger/longer unit and vice versa.)
  - *If we wanted to measure someone else with our rulers, what might be an easier way than having to count the foot lengths?*

This question is designed to lead to the idea of recording the number of units on the ruler (draw a line where the units meet and write the number at that point). Measure some more things with the class ruler. Have the students write a story about what they learned today.

Teddy Containers

Materials:

- A selection of three or four containers of different capacities (e.g., two cylindrical containers such as a tennis ball container and a cylindrical candy tin, and a rectangular prism such as a small cardboard box)
- A clear container with ten same-size, mixed-colour teddies inside (with space for them to move around) – e.g., a closed-top strawberry basket
- Coloured letter paper (about four sheets per person – some full size, some cut in half across, and some cut in half longways.) (Can use scrap letter paper instead.)
- Scissors (one pair per pair of students)
- Adhesive tape (one roll per pair of students)
- Coloured plastic teddies (all same size) – about 15–20 per pair
- Felt pens (one per pair of students) for recording on containers
- For teachers' workshop: whiteboard and pens or overhead projector and markers

Grade: 1, 2, 5 (scope to use this task up to Grade 6 with further developments)

Learner Outcomes:

Grade 1, No. 1

Demonstrate an understanding of measurement as a process of comparing by:

- identifying attributes that can be compared
- ordering objects
- making statements of comparison
- filling, covering or matching.

[C, CN, PS, R, V]

[Could also apply the following objective, but to capacity:
Grade 2, No. 3

Compare and order objects by length, height, distance around and mass (weight) using nonstandard units, and make statements of comparison.]

[C, CN, ME, R, V]

Grade 5, No. 3, 4

Demonstrate an understanding of volume by:

- selecting and justifying referents for cm^3 or m^3 units
- estimating volume by using referents for cm^3 or m^3
- measuring and recording volume (cm^3 or m^3)
- constructing rectangular prisms for a given volume.

[C, CN, ME, PS, R, V]

Demonstrate an understanding of capacity by:
- describing the relationship between mL or L
- selecting and justifying referents for mL or L units
- estimating capacity by using referents for mL or L
- measuring and recording capacity (mL or L).

[C, CN, ME, PS, R, V]

Activity Description: This activity involves problem solving related to making cylinders of a particular capacity and measuring capacity of a container using nonstandard units. Students also make comparisons between cylinders that look different but have the same capacity, thus the focus is on the notion of conservation of capacity and the language of measurement.

The Activity:

1. Introduction

- Introduce the activity by showing the container of ten teddies and moving it around so that the teddies move and make noise.
- Ask the following:
  - *What do I have here?*
  - *What do you notice?*
  - *What can you tell me about the container?*
  - *What can you tell me about the teddies?*
  - *How would you feel if you were one of the teddies? Why?* (Students may respond that they would feel sick from being shaken around!) In this discussion you want to lead the students to see that there is lots of space, i.e., the container does not hold the teddies exactly (or snugly).
  - *How many teddies do you think are in the container? How could we find out?*
- As a group, the students count teddies one by one, seeing that there are ten teddies.

2. Introduce a context

- ► *Imagine we are packers in a toy factory and have to pack ten teddies so they are nice and snug in containers. What other containers could we make? What shape might they be?*

List their ideas and then show them the different shaped containers (e.g., a small cardboard box, a small cylinder, and a larger cylinder).

– *What do we call each container?*
– *Do you think any of these containers would hold the teddies exactly?*
– *Could we make a container to hold the teddies exactly?*
– *What shape might it be?*

Ask someone to demonstrate how to roll a sheet of paper to form a cylinder (no ends required).

Are there different cylinders we can make from the same sheet of paper? (Rolled lengthwise or crosswise).

3. Making cylinders

- *Your task, in pairs, is to make a cylinder that will hold exactly ten teddies, using only a sheet of paper and tape.*
- *If you find your cylinder is too big, what might you do?* (Cut some off or roll the paper tighter.)

- Encourage them to test the capacity of their cylinder and challenge them to make any changes necessary so their teddy cylinder holds exactly ten teddies.
- Once happy, they tape the cylinder. (One student holds the paper, the other tapes the container.) Students also write “10 teddies” or “10 T” on the outside of each container that holds ten teddies.

[TEACHER WORKSHOP ONLY: MID TASK REFLECTION / DISCUSSION]

- Once a number of different looking containers have been made, stop the group (of teachers).
- **Ask:**
  – *What has been the mathematics in this activity so far?* Record on overhead projector or whiteboard. (Refer to slides if necessary.)
  – *What key ideas might we want to help students draw from all this different mathematics within this activity?*
  – *What measurement ideas?*
  – *How might we do this? (What might now happen in the lesson?)*

- This discussion is to facilitate understanding of the importance of having a clear mathematical focus and drawing this out with the students (see issues discussed in Facilitator Package), as well as the scope for further development from this one lesson.

4. Discussing the Mathematics:

- As a whole group, display the range of different teddy cylinders made. Ask questions such as:
  - *What do you notice about all the containers?*
  - *What can you tell us about them?*
  - *Find one that is different from yours. How is it different? How is it the same?*
  - *What can you say about the shape of a container and how many teddies it holds?*

- Pay attention to the language used by students in describing the containers (e.g., if someone says, "This one is bigger," ask, "*Tell me a little more,*" or "*What do you mean by bigger?*"). We want to lead the students to focus specifically on the idea of capacity, not just on a general idea of size.
- Ask the students: *"How might you sort the containers?"* You may want the students to sort them from longest to shortest. Ask: *"What do you notice?"* (e.g., the tallest container is long and skinny. The shortest container is wide.)
- *What else can you say to describe what you have found?* (e.g., containers that look different can hold the same number of teddies. In such a statement, students would be referring to the concept of conservation of capacity.)

5. Lesson Reflection

- In order to help students reflect on the focus of the lesson, a statement is written about what has been found. This might be done as a class in a large class journal or individual students might do their own recording.

Possible Extensions:

- Make a cylinder that holds twice as many teddies:
  - *I would like you to make a cylinder that holds twice as many teddies. Think about it, then describe to me what you are going to make, and then make it*.
  - *How close were you?* If it holds a different number of teddies: *Why do you think it holds more/less teddies than you thought it would?* (Note: Some students will make a container that is twice as wide and twice as high.)

Related references:

McDonough, A. (2004). "Tubs, jugs and more: Exploring volume and capacity," in B. Tadich, S. Tobias, C. Brew, B. Beatty, & P. Sullivan (Eds.), *Towards excellence in mathematics* (pp. 326–336). Mathematical Association of Victoria: Brunswick, Victoria.

McDonough, A., Cheeseman, J., & Clarke, D. (2003). "Teddy containers," in G. W. Bright & D. H. Clements (Eds.), *Classroom activities for teaching and learning measurement* (2003 Yearbook of the National Council of Teachers of Mathematics, pp. 7–9). Reston, VA: NCTM.

Calendar Challenge

Materials:
- A set of 12-month calendars (hopefully a variety of different calendars such as year at a glance, monthly, etc.)
- Copies of blackline master containing the days of the week, the numbers from 1 to 31 (see pp. 55–56)
- Copies of the recording sheet Similar/Different see p. 53)
- Blank paper
- Glue
- Scissors

Grade: 2

Learner Outcomes: Grade 2, No. 1
Relate the number of days to a week and the number of months to a year in a problem solving context.
[C, CN, PS, R]

Activity Description: This activity focuses on the concept of time and the sequencing of days of the week and months of the year. Students are also asked to relate common activities to specific units of time.

The Activity:

1. Introduction

- Students are asked to identify a day that is special to them. They should know the date of that day (e.g., their birthday or name day). If necessary, students can collect and record this information for homework.
- On a classroom calendar, students can present the date of their special day and show where it is on the calendar.

Note: Some students may need help with this.

2. Same and Different

- Put students in groups of two to four.
- Provide each group with a 12-month calendar. Ask students to look at the calendar for each of the 12 months and find similarities and differences among the monthly calendars.
- For example, they may say each month had the seven days of week in order but the first day does not always fall on the same day.
- Students record their findings on their Same/Different Chart.

How calendar months are…	
Similar	Different
____________	____________
____________	____________
____________	____________
____________	____________

- Students share their findings with the class. You may develop a class same/different chart for recording their findings.
- Discuss to lead students to the conclusions:
  - There are seven days in a week and the days follow a given order.
  - Each calendar for the year has 12 months.
  - There are 30 or 31 days in a month except for February.
  - Every month had three full weeks.
  - The months of the year occur in order (January, February, March, April, May, June, July, August, September, October, November, December).
  - There are patterns in the numbers on a calendar. For example, as you go down the column for a given day of the week, the numbers go up by seven. (Discuss why that is so.)
  - Some calendars start with Monday and others start with Sunday.

3. Making a Calendar:

- Provide students with blackline masters, pp. 55–56. Have the students cut out the days of the week and the number cards from 1 to 28, 30, or 31. (Student can choose.)
- Have students create a monthly calendar. The student can choose which month to make and write the name at the top of the calendar. Students can also choose which day of the week the first day of the month will fall on.
  Note: The number of days in the month must match the name for the month.
- Have the student add events that might occur during this month. For example, in the month of February they may wish to add Valentine's Day. They may also wish to add events such as hockey on the days they usually go to hockey.
- As you walk around the room, notice if the students put the days of the week in order, the numbers in order, etc.
- Ask the following:
  - *What is the name of the month following the month of your calendar?*
  - *What day will the first day of the month fall on in the next month? Explain how you know.*
  - *How many full weeks are in your month and how many days are left over? Draw a box around the full weeks.*
  - *On what dates will Fridays fall during this month?*
  - *What day is the 19th of the month?*

- Try to make a year calendar by having students come up to the front and stand, one behind the other, with their monthly calendars. (Start with a student who made a January calendar. Behind him will stand a student who made a February calendar, etc. If months are missing, write that month's name on a blank sheet of paper and have someone hold it as a place holder.)

  Discuss the following questions:

  – *Which months have 31 days?*
  – *What month comes after June?*
  – *What is the last month of the year?*
  – *What are the names of the months in order?*

4. **Practising the months of the year/days of the week:**
   - Play the game Sit Down December, Sit Down Saturday. In this game, the students stand in a circle and recite months of the year, sitting down and missing one turn after they say a nominated month. They then rejoin the game.
   - Play the same game but with the months of the year (or days of the week) in reverse order, thus focusing on the month (or day) before.
   - Teach the students the following poem:

     Thirty days hath September
     April, June, and November.
     All the rest have thirty-one.
     Except for February which has twenty-eight
     But Leap Year comes once in four
     February then has one day more.

   - Ordering months and days
     – Students find their special day on the calendar for the year and then each write the date for their special day on a strip of paper (e.g., Friday, July 14). The students then, holding their strips of paper, silently order themselves by month and day. Discussion can include identification of any missing months (and any missing days). Attention is brought to the order of the months.

Extension:

- Discuss the seasons as they relate to months of the year. You may want to use the following questions:

  – *How many seasons are there in a year? What are they?*
  – *Which months are the winter months?*
  – *What season is it during July?*
  – *What season comes after summer?*
  – *During which season is Christmas?*
  – *During which season is your birthday?*

Month: ____________________ **Year:** _________

THURSDAY	SUNDAY		WEDNESDAY		FRIDAY	TUESDAY
16	22	3	4	25	6	17
28	9	7	18	23	13	14
15	1	10	11	19	27	21
2	12	30	5	26	20	8
	24	31	29	MONDAY		SATURDAY

Guess and Check (Seconds and Minutes)

Materials:
- Stopwatch or clock with a second hand
- Guess and Check recording sheets (see p. 59)
- Unifix cubes and sticks for extension

Grade: 3

Learner Outcomes:

Grade 3, No. 1
Relate the passage of time to common activities using nonstandard and standard units (minutes, hours, days, months and years).
[CN, ME, R]

Grade 3, No. 2
Relate the number of seconds to a minute, the number of minutes to an hour and the number of days to a month in a problem-solving context.
[C, CN, ME, PS, R, V]

Activity Description: This activity focuses on the concept of time, considering units of time, duration of time and passage of time. Students make estimates of how much of a particular activity they can do in 15 seconds, 30 seconds or one minute and check their estimates by undertaking the activity.

The Activity:

Choose a selection from the activities listed on the sheet, e.g., blinking eyes in 15 seconds, foot tapping in 15 seconds, foot tapping in 30 seconds, naming foods in one minute. Complete these one at a time, with the teacher conducting the timing.

1. Introduction:

- To clarify this activity, ask the students, "*If I was to tell you to blink a few times, what might I mean? What is a blink?*"

2. Students Estimate and Test:

- Tell students you are going to ask them to blink for 15 seconds. Ask student to think about how many times they could blink in 15 seconds, then, *"Write down your estimate of how many times you think you will blink in 15 seconds. When I start the stop watch you are to start blinking and counting your blinks."* Time 15 seconds for the students using a stopwatch.

Have students write down the number of blinks made in 15 seconds.

- *How many times do you think you will be able to tap your foot in 15 seconds?*

Students record their estimate in the first column.

Students test and record, in the second column, their number of foot taps for 15 seconds.

Ask: *"Did you do the same number of foot taps as blinks?*" If different, *"Why do you think that was so?"*

- Have students estimate, record and test for foot tapping for 30 seconds.

- Discuss:
  - *How close were you to your estimate?*
  - *What strategy did you use to make your estimate?* (If relevant: *Why do you think the number was more/less than what you estimated?*)

One student in each pair undertakes saying his or her own name for one minute. Partner counts. Students records count after finishing.

Partners then swap roles.

How did you do?

- To help students understand that perception of time depends upon what activity you are doing, ask: *"Did the time seem to go more slowly for any of the activities?"* (For many, foot tapping seems to take longer as it can be tiring).

Extensions:

- *How many Unifix can you put in a stick in one minute?* (Students could compare this visually by making a graph.)
- Have students use a personal referent such as taping their foot, saying their name or counting by 1's to estimate 30 seconds.
  - Students choose referent to use.
  - Teacher says *"when I say start, I want you to time 30 seconds. When you think 30 seconds is up, show me a 'thumbs up.'"*
  - Teacher times 30 seconds but doesn't say stop until 45 seconds are up. Note which students were close to 30 seconds with their estimate and which were not.
  - Have students share strategies.
  - Repeat, telling students to stop at the 30-second mark. Again discuss strategies for refining their estimates.
  - Provide opportunities to practise estimating time throughout the year.

Guess and Check

In 15 seconds	Guess	Check
How many times can you blink your eyes?		
How many times can you tap your foot?		
How many Unifix can you put in a stick?		

In 30 seconds	Guess	Check
How many times can you blink your eyes?		
How many times can you tap your foot?		
How many Unifix can you put in a stick?		

In one minute	Guess	Check
How many different foods can you name?		
How many times can you tap your foot?		
Counting by ones, what number can you count to?		
How many times can you say your full name?		

How Long Does It Take?

Materials:
- Chart paper and markers
- Stopwatch or clock with a second hand
- Copy of Activities Cards (see pp. 63–64) that have been copied and cut out. One per pair or group of students

Grade: 3

Learner Outcomes:

Grade 3, No. 1
Relate the passage of time to common activities using nonstandard and standard units (minutes, hours, days, months and years).
[CN, ME, R]

Grade 3, No. 2
Relate the number of seconds to a minute, the number of minutes to an hour and the number of days to a month in a problem-solving context.
[C, CN, PS, R, V]

Activity Description: This activity focuses on the concept of time, considering units of time, duration of time and passage of time. Students are asked to select personal referents for seconds and minutes and to associate common activities with specific units of time.

The Activity:

1. Seconds and Minutes

- On chart paper write the titles: Seconds, Minutes.
- To clarify what a second is, have students look at the clock and sit perfectly still for one second.
- Then have students brainstorm events that might last one second. Record these on the chart paper under the title seconds.
- To clarify what a minute is, have the students look at the clock and sit perfectly still for one minute. Discuss: *Is it longer or shorter than a second? A lot longer or just a bit longer?* Try to establish the fact that a minute is quite a bit longer than a second. Then tell students a minute is equal to 60 seconds.

Have students look at the worksheet “Guess and Check.” *I am going to time one minute and you will do each of the following activities for the one minute:*

- *tap your foot*
- *count by ones as high as you can*
- *say your name as many times as possible*

Have students estimate and record their estimate. Then complete the task and record the results on the recording sheet.

- Brainstorm events that last one minute and record on chart paper.

Extension:

- Some dentists say you should brush your teeth for two full minutes to do a really good job of cleaning them. Have students imagine that they are brushing their teeth and time two minutes. This will give students a feel for how long two minutes really is.

2. Estimating Time Using Referents:

- *If I was to ask you to time one minute, what might you use if you did not have a clock?* Brainstorm ways to determine the passage of one minute. Have students select a referent to use.
- Have students close their eyes and when they think one minute is up, raise their thumb. The teacher times one minute and notes those students who are able to time close to a minute. This gives feedback on how students are using their personal strategy for timing a minute.
- Pair students up with a partner. One partner has a stopwatch or can see the second hand on an analogue clock. Ask students to try to time one minute from the time their partner says “Go” by using a personal strategy. The partner will time the student that is estimating a minute and tell them how close to a minute they were. Switch roles and repeat the activity.

3. Hour, Day and Month

- Write the titles “Hour” and “Day” on chart paper.

  - Have students brainstorm events lasting one hour and explain a sequence of events that would last one day. Discuss the relationship between the passage of an hour and a day.
  - Establish the fact that there are 24 hours in one day.

Extension:

- Ask students what events last about half an hour. Relate this to one hour.
- Refer to the title Month.
  - Have students brainstorm events that can be completed in one month. Discuss the relationship between the passage of one day and one month.
  - Establish the fact that there are about 30 days in a month. (Refer to calendar lesson.)

4. Which Unit of Time Should I Use?

- Provide the students with copies of blackline master (p. 63) or create your own using activities that are relevant to your community.
- Students cut out the activity cards (see p. 63) and the title cards for Hour, Second, Minute.
- Students work in small groups. The pile of activity cards is turned face down. Students draw a card from this pile and decide as a group whether it should go under the Hour, Minute or Second title in terms of what units of time are best used to describe the duration of this activity.

Note: For some activities, there may be more than one correct answer as long as students can justify their answer.

Activities

Watch your favourite TV show	Brush your teeth	Tie your shoes
Travel to school from home	Eat lunch	Play a game of hockey
Run once around the playground	Do 10 sit-ups	Count to 100
Write your name 20 times	Read a book	Sleep through the night
Say your name 20 times	Build a snowman	Watch a movie
Build a tower of 50 unifix cubes	Snap your fingers 15 times	Blink slowly five times
Run across the gym and back	Make a sandwich	Take a bath

HOUR	**MINUTE**	**SECOND**

Mass Activities: Nonstandard Units

Materials:

- Handout of recording chart (p. 66)
- Blackline masters (see pp. 69–70)
- Pan balance for each group
- Collection of objects of varying mass
- Overhead (p. 68)
- Unifix cubes
- Play dough

> Play Dough Recipe
>
> Mix:
> 2 cups flour
> 1 cup salt
> 2 tablespoons cooking oil
> Water
> Food colouring
>
> Combine ingredients, using just enough water to allow the mixture to be kneaded. Knead until smooth. Store in an airtight container.

Grade: 2

Learner Outcomes: Grade 2, No. 3
Compare and order objects by length, height, distance around, mass (weight) using nonstandard units, and make statements of comparison.
[C, CN, ME, R, V]

Activity Description: These activities all focus on the attribute of mass. They are hands-on activities, using familiar objects and nonstandard units.

The Activity:

1. Introduction:

- Have students discuss what it means to find the mass of the object. Ask,
  - "*What objects have you seen being weighed?*"
  - *"Why might we need to know the mass (weight) of an object?"* List responses on the board.

- Show students a pan balance and demonstrate how it works. Ask,
  - *"What does the pan balance do?"*
  - *"How does it work?"* (You may want to refer to the teeter-totter.)
- Have students place objects on the balance and have the class answer.
  - *"Which object is lighter?"*
  - *"Which is heavier?"*
  - *"How do you know?"*
- You may want to summarize their finding in a diagram. See overhead master (p. 68).

2. Compare and Order Objects by Mass

- Provide groups of students with a collection of objects of varying mass and a pan balance. Have a student pick up two objects and predict which is heavier by holding one object in each hand. Have another student verify their prediction by using a pan balance. Have students take turns doing this, recording the results by filling out the chart below:

Objects	I predicted this object was heavier	I found out that this object was heavier
Rubber ball and a can of pop	Can of pop	Can of pop
Pencil and Unifix cube stack	Unifix cube stack	pencil

- Have students make statements of comparison about the objects. Share these with the class, showing the objects as they do so.
- Have the students then try to order several objects from lightest to heaviest.
- Discuss by asking questions such as the following:
  - *"Does the larger the size of the object mean it has the greatest mass?* " (Provide an example to justify your answer.)
  - *"Can two different objects have the same mass?"*
- Complete blackline master (p. 69) "Comparing Mass."

3. Conservation of Mass:

- Make two balls of play dough that have the same mass. Establish that the mass of the two balls is the same by using a pan balance and having the students see that the arms of the pan balance are even when one ball is placed on each pan.

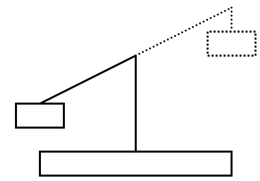
- Remove the balls from the pan balance. Change the shape of one of the balls by flattening it out. Then ask,
  - *"Do the two pieces of play dough have the same mass or is one piece heavier?"*
  - *"How do you know?"*
- Use the pan balance to prove or disprove the predictions.
- Form three different play dough shapes that each have the same mass. Label the shapes A, B, and C. Place A and B on the pan balance so the students agree that A and B balance each other. Then place B and C on the pan balance so that the students agree that B and C balance each other. Lastly ask,
  - *"Do A and C have the same mass or is one of them heavier?"*
  - *"How do you know?"*
- As a follow up to this, you may want to provide students with a lump of play dough and have them break it into two lumps of equal mass. Students may use pan balances to verify this. Have the students then take one of the two equal lumps of play dough and mold it into a model. Ask,
  - *"Which is heavier, the lump of play dough or your model?"* Verify their prediction using a pan balance.

4. <u>Using Nonstandard Units:</u>

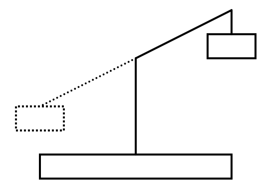
- Place a ball of play dough on one side of a pan balance and ask students, "*How many Unifix cubes will it take to balance the pan balance?"* Write their estimates on the board. Ask, *"How can we check to see who has the closest estimate?"* Discuss then verify the mass of the ball of play dough by placing Unifix cubes in the pan balance until it is balanced.
- Tell students they are going to estimate how many Unifix cubes are needed to match the mass of a set of objects. Provide copies of blackline master "How Many Blocks?" (p. 70). Have students complete the activity.
- Discuss the following:
  - *Which object was the heaviest? The lightest? How do you know?*
  - *If we used pennies in place of Unifix cubes, would we still find the same object to be the heaviest?* Discuss then verify.
  - *What objects might they have used in place of the Unifix cubes?*

Be a Mass Master

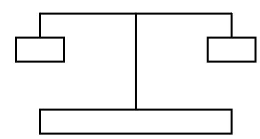
Heavier



Lighter



Same



Comparing Mass

You will need: classroom objects, balance scales

- Work with a partner.
- Choose an object to go in the centre. Draw it.
- Collect and record classroom objects that fit into the four groups on the table.

smaller and heavier	smaller and lighter
Larger and heavier	larger and lighter

our object

- Did you use lifting or balance scales to measure?

  ____

- Why did you use this way to measure?

  ____

How Many Blocks?

You will need: the objects listed below, balance scales, Unifix, colouring materials

- Estimate (guess) how many Unifix you need to balance each object. Colour the graph.
- Place each object on the balance scales.
- Colour the graph to record how many Unifix blocks you need to balance each object.

Pencil	Estimate (Guess)								
	Measure (Check)								
Package of crayons	Estimate (Guess)								
	Measure (Check)								
Scissors	Estimate (Guess)								
	Measure (Check)								
Small stone	Estimate (Guess)								
	Measure (Check)								
Book	Estimate (Guess)								
	Measure (Check)								
Your choice	Estimate (Guess)								
	Measure (Check)								

- Which object was the heaviest? Lightest?

  ____

Reproduced with permission from Sandy Woodcock, Marilyn Doney and Candace Pacey, *Math Works: A Balanced Approach to Teaching Mathematics, Grade 2 Teacher's Resource* (Markham, ON: Scholastic Canada Ltd., 2004), p. 155.

More Mass Activities

Materials:

Mass Activity 1:
- one of each: potato, onion, orange, grapefruit, banana, apple
- balance scales
- plastic teddies
- blackline master (p. 73)

Mass Activity 2:
- one kilogram packet of sugar
- balance scales
- formal mass pieces (weights)
- blackline master (p. 74)

Mass Activity 3:
- a piece of packaging foam (larger but lighter than each of the other two items)
- a small box such as a matchbox (with some plasticine inside)
- a pair of scissors
- balance scales
- centicubes
- blackline master (p. 75)

Mass Activity 4:
- balance scales
- formal mass pieces (weights)
- blackline master (p. 76)

Grade: 2, 3

Learner Outcomes:

Grade 2, No. 3
Compare and order objects by length, height, distance around, mass (weight) using nonstandard units, and make statements of comparison.
[C, CN, ME, R, V]

Grade 3, No. 4
Demonstrate an understanding of measuring mass (g and kg) by:
- selecting and justifying referents for the units g and kg
- modelling and describing the relationship between the units g and kg
- estimating mass using referents
- measuring and recording mass.

[C, CN, ME, PS, R, V]

* I acknowledge my colleague Ann Downton for the idea of using different foods for children's measuring within these mass activities.

Activity Description: These activities all focus on the attribute of mass. They are hands-on activities, using familiar objects and either nonstandard or standard units. Estimating by hand (hefting), comparing and ordering are also features of the activities. The use of an object such as one kilogram packet of sugar can also help students develop benchmarks for mass. The intention is that such activities are used with the class broken into groups and rotating through the activities. By having all activities concerned with the attribute of mass, and ideally with a more specific common focus such as using nonstandard units or developing benchmarks, a class review can be conducted that is meaningful to all students and to which all can potentially contribute.

MASS ACTIVITY 1

Your group has a potato, an onion, an orange, a grapefruit, a banana, and an apple.

BY HAND

Which feels the heaviest? Which feels the lightest? Place the food in order from least to most mass.

Record the order.

USING THE BALANCE SCALES

Check the order using the balance scales.

Record the order.

WHAT IS THE MASS OF EACH IN TEDDIES?

Estimate the mass of each piece of food in teddies, by hefting. Record your estimates.

Check using the balance scales and teddies.

Materials required: potato, onion, orange, grapefruit, banana, apple, balance scales and teddies.

MASS ACTIVITY 2

WHAT HAS THE SAME MASS AS ONE KILOGRAM OF SUGAR?

ESTIMATE AND USE THE BALANCE SCALES TO CHECK.
What is one objects in the room that has the same mass as the sugar?

What are two different objects that together have the same mass as the sugar?

What are three different objects that together have the same mass as the sugar?

ESTIMATE
Estimate the mass of each object. Record your estimates.

MEASURE
Find the mass of each object by using the pan balance and mass pieces. Record what they weigh.

How close were your estimates?

Materials required: one kilogram packet of sugar, balance scales and formal mass pieces (weights).

MASS ACTIVITY 3

You have three articles: a piece of foam, a matchbox and a pair of scissors.

BY HAND

Which of the three objects feels the heaviest?

Which feels the lightest?

USING THE BALANCE SCALES

Use the balance scales to put them in order from heaviest to lightest.

Find the mass of each object by using the centicubes (each centicube weighs one gram).

Materials required: a piece of packaging foam, a small box, a pair of scissors, balance scales, centicubes.

MASS ACTIVITY 4

WHAT DOES TWO KILOGRAMS FEEL LIKE?

BY HAND

Find a collection of objects that you believe has a total mass of two kilograms.

Estimate the mass of each item. Record your estimates.

USING THE BALANCE SCALES

Use the pan balance and mass pieces to find the total mass of the collection and the mass of each item. Record the mass of each item and the total.

How close were your estimates? How close to two kilograms were you?

What did you use to help you to make your estimates?

Materials required: balance scales and formal mass pieces (weights).

Mouse House

Materials:

- Origami squares 10 cm x 10 cm (12 per pair of students) – with extra tiles available
- Newspapers (about two sheets per student)
- Adhesive tape available
- One felt pen per student
- Scissors (one pair per pair of students)
- Grid paper – available for possible use
- Rulers and tape measures
- Overhead projector
- Square tiles

Grade: 3

Learner Outcomes: Grade 3, No. 5

Demonstrate an understanding of perimeter of regular and irregular shapes by:

- estimating and recording perimeter (cm and m)
- constructing different shapes for a given perimeter (cm, m) to demonstrate that many shapes are possible for a perimeter.

[C, ME, PS, R, V]

Activity Description: The attribute of length is applied here as perimeter. Students arrange 12 floor tiles to make different mouse houses for a pet mouse. The floors of these houses might be rectangular or some irregular shape. The students then measure the perimeter of each floor, cut out the floor and record the perimeter. This allows discussion of what they think might be the best mouse house.

The Activity:

1. Introduction:

- *Does anyone have a room at home that has tiles on the floor?*
- *What do these look like?*
- *How are they arranged?* Note that some tiles have spaces between them for grout but that some others, such as carpet tiles, do not. We have some tiles that do not need spaces between them.

- *We have been given the job of constructing houses for pet mice!*
  - *If a mouse house was to have six tiles on the floor, what might the floor look like? Show me using these tiles on the overhead.*
- *What can you say about the floor of this mouse house?*
- *Let's make a different mouse house that has a floor of six tiles* (next to the first one made).
  - *What can you say about this mouse house floor?*
  - *How are the two floors we have made the same?*
  - *How are they different?*
- *If we were to have a wall go around the outside each of these mouse houses,*
  - *How much wall would we need?*
  - *How could we find out? (Possible suggestions are to count the sides of the origami squares – but be sure that students count the perimeter or measure with a ruler or tape measure.)*
- *Could we have a mouse house with six floor tiles that looks different again?* (They do not have to be rectangular).
  - *How much wall would be needed?* (Walls come in unit pieces, each the same length as one side of a square.)
- *So that we can compare the different possibilities, we will make a model of each mouse house. To do this we will trace the floor into newspaper, cut it out and write the amount of fence needed. This will help us make a final decision of the mouse house we will build.*

2. Students Investigate

- *You are have been given the job of constructing a house for a pet mouse. You have been given 12 tiles that can be used as a floor in your mouse house.*
- *Arrange the tiles into one possible shape that could be a house for your mouse. Then explore other possible shapes. (Make at least two).*
- *For each of these floors find out the amount of wall that would be needed.*
- *For each Mouse House floor that you make, trace it onto newspaper, cut out the shape, and record the perimeter.*
- Students work in pairs to explore, make and record.

3. Sharing and Discussion

- Students share some of their mouse house floors, showing the newspaper cut-outs to demonstrate what is possible.

- Within the discussion, challenge them to decide which they think would be the best house for a pet mouse. (They might consider the most comfortable mouse run, the cost of materials, etc.)
- *We have been exploring the amount of wall that is needed to go around each of these floors.*
  - *What is the mathematical name for the distance around?* (Perimeter).
  - *Did you find any short cuts for finding the perimeter of any of the shapes?* (e.g., for a regular shape finding the perimeter of two sides and doubling it).

4. Challenge

- *Choose a particular perimeter and make as many different shaped floors as you can for that perimeter* (these might be regular or irregular). *Be ready to describe your perimeter and what different-shaped floors are possible for this perimeter.*

5. Reflection

- *You are to make a record statement and have examples to demonstrate to others what you have found. You are to choose a way to do this.* (Have grid paper available for possible use.)

- Sharing and discussion. Students share what they have found.
- Teacher question:
  - *What can we say to summarize what we have found today?* (e.g., there are many possible shapes for the same perimeter).

Extensions:

1. Investigation:

- Would a mouse house made with 24 tiles need twice the amount of wall as of those with 12 tiles?

Cooperative Problem Solving: Perimeter

Materials:
- Blackline masters (pp. 81–84)
- Pattern blocks
- One set of task cards for each group of four students

Grade: 3

Learner Outcomes: Grade 3, No. 5

Demonstrate an understanding of perimeter of regular and irregular shapes by:
- estimating and recording perimeter (cm and m)
- constructing different shapes for a given perimeter (cm, m) to demonstrate that many shapes are possible for a perimeter.

[C, ME, PS, R, V]

Activity Description: Students work in groups of four to solve a perimeter problem using pattern blocks. Students really enjoy this activity and it helps consolidate their understanding of perimeter.

The Activity:

- Students are placed in groups of four.
- Each student has one task card. They read their card to the group. The group must find the solution to the problem by creating a shape that meets the criteria on each of the four task cards. The student holding the task card must ensure that his or her criteria has been met in the final solution. It is important that the group work together to solve the problem.

Perimeter

Make a shape with a perimeter of 7 units. Use 4 blocks. **36**	Use 3 different colours of blocks. No block is orange. **36**
Two blocks are the same colour. They are not red. **36**	One short side of the red trapezoid equals 1 unit of perimeter. **36**

Perimeter

Make a shape with a perimeter of 8 units. **36**	Use 6 blocks. **36**
Use only 1 orange square. Each of its sides equals 1 unit of perimeter. **36**	One short side of the red trapezoid equals 1 unit of perimeter. **36**

Perimeter

Make a shape with a perimeter of 7 units.

33

Use 1 yellow block. One of its sides equals 1 unit of perimeter.

33

Use fewer than 4 blocks.

33

Use blocks of 2 different colours.

33

Reproduced from Ann Roper, *Cooperative Problem Solving with Pattern Blocks* (Mountain View, CA: Creative Publications, 1989), p. 33. Reproduced with the permission of The McGraw-Hill Companies.

Perimeter

Make a rectangle with a perimeter of 20 units.

34

Put the orange blocks in rows. Make fewer than 5 rows.

34

Use all orange blocks.

34

One side of an orange block equals 1 unit of perimeter.

34

Reproduced from Ann Roper, *Cooperative Problem Solving with Pattern Blocks* (Mountain View, CA: Creative Publications, 1989), p. 34. Reproduced with the permission of The McGraw-Hill Companies.

Conservation of Perimeter

Materials:
- Strings, each 60 cm long (each of the five figures from the blackline master will be matched with a different string)
- Corrugated cardboard or manila tag

Grade: 2, 3

Learner Outcomes:

Grade 2, No. 3

Compare and order objects by length, height, distance around and mass (weight) using nonstandard units, and make statement of comparison.
[C, CN, ME, R, V]

Grade 3, No. 5

Demonstrate an understanding of perimeter of regular and irregular shapes by:
– estimating and recording perimeter (cm and m)
– constructing different shapes for a given perimeter (cm, m) to demonstrate that many shapes are possible for a perimeter.
[C, ME, PS, R, V]

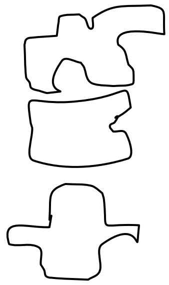
Activity Description: In this activity students will find and compare the perimeter of irregular shapes to discover that many shapes can have the same perimeter.

The Activity:

1. Conservation of Perimeter:

- Give the students a string ten to thirty centimetres long. Have them make a closed shape with their string and trace it. (Hint: Trace the outside edge of the shape with dotted lines so the string doesn't move.)
- Ask the students to create other shapes by using the same piece of string. Through discussion, establish that figures of various shapes may have the same distance around (perimeter).
- Have them record the results. This visually shows that the perimeter is the same for figures of different shapes. (See example.)

Sample of student recording:



Perimeter of Shapes

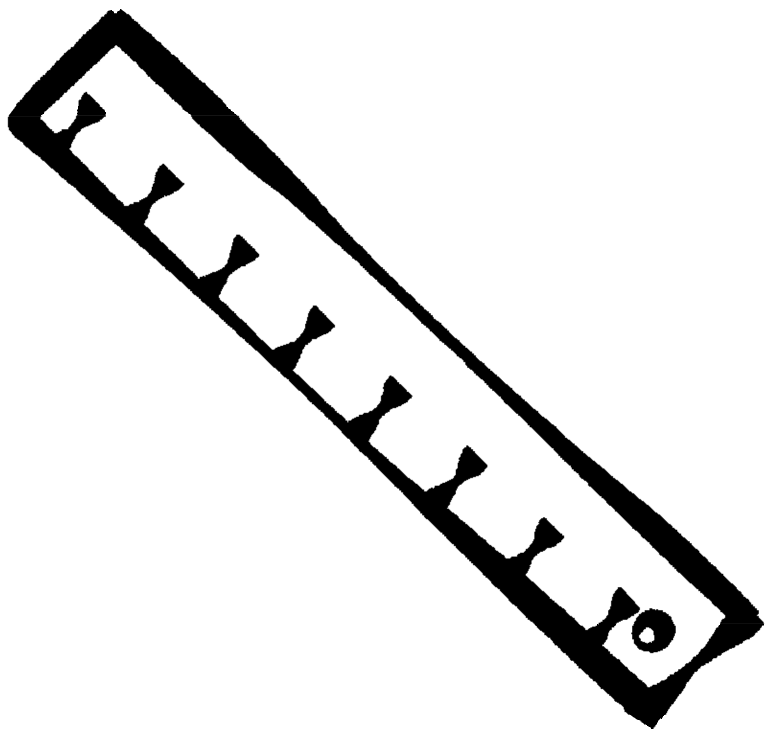
35 cm

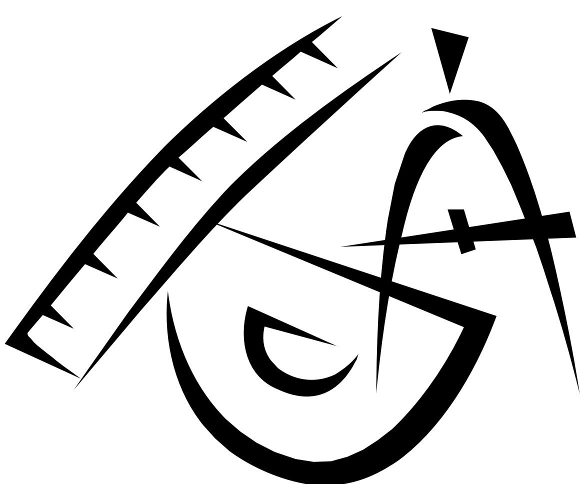
35 cm

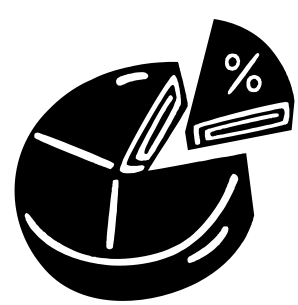
35 cm

- Have students write a sentence describing what they have learned about perimeter.

Recording Sheets







A BAG OF APPLES

Brief description: Students choose something in the room that they estimate will be the same length as their apple peel (when peeled). They peel their apple (it may be in pieces) and compare the length to other things in the room by using a third object such as a piece of string.

What is the mathematics?

What do we want students to notice?

Teacher role / questioning:

Features of the activity:

Adapt / extend / make simpler:

EXTRA NOTES:

WOOLLY WORMS

Brief description: Students are introduced to the concept of woolly worms. In pairs, they each make a bag of five woolly worms that are all different in length.

What is the mathematics?

What do we want students to notice?

Teacher role / questioning:

Features of the activity:

Adapt / extend / make simpler:

EXTRA NOTES:

HOW LONG IS YOUR STRING?

Brief description: Students use pieces of string to estimate length. This activity develops the use of comparative language.

What is the mathematics?

What do we want students to notice?

Teacher role / questioning:

Features of the activity:

Adapt / extend / make simpler:

EXTRA NOTES:

MAKE A TEDDY RULER

Brief description: Students are to make a ruler that is the same length as ten teddies. They measure objects with this ruler.

What is the mathematics?

What do we want students to notice?

Teacher role / questioning:

Features of the activity:

Adapt / extend / make simpler:

EXTRA NOTES:

HOW BIG IS A FOOT?

Brief description: This activity is designed to have students think about the need for using standard units of measure.

What is the mathematics?

What do we want students to notice?

Teacher role / questioning:

Features of the activity:

Adapt / extend / make simpler:

EXTRA NOTES:

TEDDY CONTAINERS

Brief description: Working in pairs, students make a range of different cylinders that will each hold exactly ten teddies.

What is the mathematics?

What do we want students to notice?

Teacher role / questioning:

Features of the activity:

Adapt / extend / make simpler:

EXTRA NOTES:

CALENDAR CHALLENGE

Brief description: This activity focuses on the concept of time and the sequencing of days of the week and months of the year. Students relate common activities to specific units of time.

What is the mathematics?

What do we want students to notice?

Teacher role / questioning:

Features of the activity:

Adapt / extend / make simpler:

EXTRA NOTES:

GUESS AND CHECK

Brief description: Students estimate the number of times they can do a particular action in 15 seconds, 30 seconds or one minute, and then test.

What is the mathematics?

What do we want students to notice?

Teacher role / questioning:

Features of the activity:

Adapt / extend / make simpler:

EXTRA NOTES:

HOW LONG DOES IT TAKE?

Brief description: This activity focuses on the concept of time. Students develop and select personal referents for seconds and minutes and associate common activities with specific units of time.

What is the mathematics?

What do we want students to notice?

Teacher role / questioning:

Features of the activity:

Adapt / extend / make simpler:

EXTRA NOTES:

MASS ACTIVITIES: NONSTANDARD UNITS

Brief description: Students compare and order the mass of objects using nonstandard units.

What is the mathematics?

What do we want students to notice?

Teacher role / questioning:

Features of the activity:

Adapt / extend / make simpler:

EXTRA NOTES:

MORE MASS ACTIVITIES

Brief description: In groups, students complete a range of activities that focus on the attribute of mass. They are hands-on activities, using familiar objects and either nonstandard or standard units. Estimating (by hefting), comparing and ordering are also features of the activities.

What is the mathematics?

What do we want students to notice?

Teacher role / questioning:

Features of the activity:

Adapt / extend / make simpler:

EXTRA NOTES:

MOUSE HOUSE

Brief description: Students create and compare floors for mouse houses each having the same number of tiles. They investigate perimeters (fencing) for mouse houses.

What is the mathematics?

What do we want students to notice?

Teacher role / questioning:

Features of the activity:

Adapt / extend / make simpler:

EXTRA NOTES:

COOPERATIVE PROBLEM SOLVING

Brief description: Students work in groups of four to solve a perimeter problem using pattern blocks.

What is the mathematics?

What do we want students to notice?

Teacher role / questioning:

Features of the activity:

Adapt / extend / make simpler:

EXTRA NOTES:

CONSERVATION OF PERIMETER

Brief description: In this activity, students will find and compare the perimeter of irregular shapes to discover that many shapes can have the same perimeter.

What is the mathematics?

What do we want students to notice?

Teacher role / questioning:

Features of the activity:

Adapt / extend / make simpler:

EXTRA NOTES:

Reflecting on the day:

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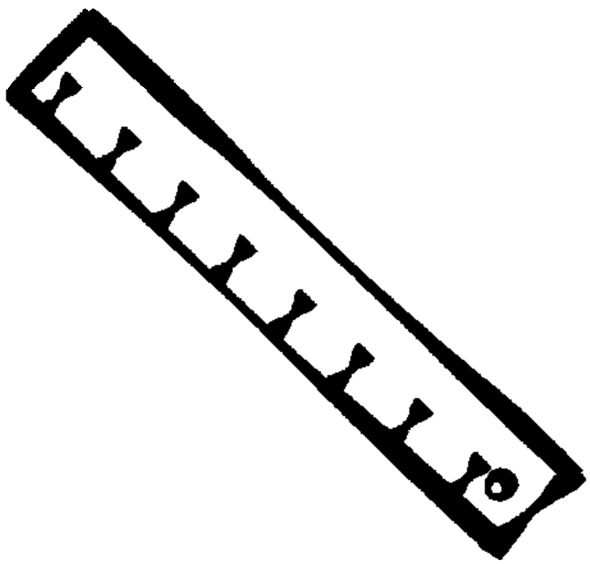
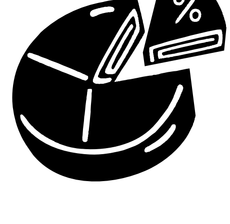
Western and Northern Canadian Protocol Common Curriculum Framework for Mathematics (WNCP CCF)





[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[ME]	Mental Mathematics and Estimation	**[T]**	Technology
		[V]	Visualization

Kindergarten **Strand:** Shape and Space (Measurement)	**General Outcome:** Use direct or indirect measurement to solve problems.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
1. • Use direct comparison to compare two objects based on a single attribute, such as length, weight and capacity. [C, CN, PS, R, V]	➢ Compare the length (height) of two given objects and explain the comparison using the words shorter, longer (taller) or almost the same. ➢ Compare the mass (weight) of two given objects and explain the comparison using the words lighter, heavier or almost the same. ➢ Compare the volume (capacity) of two given objects and explain the comparison using the words less, more, bigger, smaller or almost the same.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[ME]	Mental Mathematics and Estimation	**[T]**	Technology
		[V]	Visualization

Grade 1 **Strand:** Shape and Space (Measurement)	**General Outcome:** Use direct or indirect measurement to solve problems.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators **may** be used to determine whether students have met the corresponding specific outcome.*
1. Demonstrate an understanding of measurement as a process of comparing by: • identifying attributes that can be compared • ordering objects • making statements of comparison • filling, covering or matching. [C, CN, PS, R, V]	➢ Identify common attributes, such as length (height), mass (weight), volume (capacity) and area, that could be used to compare a given set of two objects. ➢ Compare two given objects and identify the attributes used to compare. ➢ Determine which of two or more given objects is longest/shortest by matching and explain the reasoning. ➢ Determine which of two or more given objects is heaviest/lightest by comparing and explain the reasoning. ➢ Determine which of two or more given objects holds the most/least by filling and explain the reasoning. ➢ Determine which of two or more given objects has the greatest/least area by covering and explain the reasoning.

[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[ME]	Mental Mathematics and Estimation	**[T]**	Technology
		[V]	Visualization

Grade 2 **Strand:** Shape and Space (Measurement)	**General Outcome:** Use direct or indirect measurement to solve problems.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators* ***may*** *be used to determine whether students have met the corresponding specific outcome.*
1. Relate the number of days to a week and the number of months to a year in a problem-solving context. [C, CN, PS, R]	➢ Read a date on a calendar. ➢ Name and order the days of the week. ➢ Identify the day of the week and the month of the year for an identified calendar date. ➢ Communicate that there are seven days in a week and twelve months in a year. ➢ Determine whether a given set of days is more or less than a week. ➢ Identify yesterday's/tomorrow's date. ➢ Identify the month that comes before and the month that comes after a given month. ➢ Name and order the months of the year. ➢ Solve a given problem involving time which is limited to the number of days in a week and the number of months in a year.
2. Relate the size of a unit of measure to the number of units (limited to non-standard units) used to measure length and mass (weight). [C, CN, ME, R, V]	➢ Explain why one of two given non-standard units may be a better choice for measuring the length of an object. ➢ Explain why one of two given non-standard units may be a better choice for measuring the mass of an object. ➢ Select a non-standard unit for measuring the length or mass of an object and explain why it was chosen. ➢ Estimate the number of non-standard units needed for a given measurement task. ➢ Explain why the number of units of a measurement will vary depending upon the unit of measure used.
3. Compare and order objects by length, height, distance around and mass (weight) using non-standard units, and make statements of comparison. [C, CN, ME, R, V]	➢ Estimate, measure and record the length, height, distance around or mass (weight) of a given object using non-standard units. ➢ Compare and order the measure of two or more objects in ascending or descending order and explain the method of ordering.

[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Grade 2 **Strand:** Shape and Space (Measurement) (continued)	**General Outcome: Use direct or indirect measurement to solve problems.**
4. Measure length to the nearest non-standard unit by: • using multiple copies of a unit • using a single copy of a unit (iteration process). [C, ME, R, V]	➢ Explain why overlapping or leaving gaps does not result in accurate measures. ➢ Count the number of non-standard units required to measure the length of a given object using a single copy or multiple copies of a unit. ➢ Estimate and measure a given object using multiple copies of a non-standard unit and using a single copy of the same unit many times, and explain the results. ➢ Estimate and measure, using non-standard units, a given length that is not a straight line.
5. Demonstrate that changing the orientation of an object does not alter the measurements of its attributes. [C, R, V]	➢ Measure a given object, change the orientation, re-measure and explain the results.

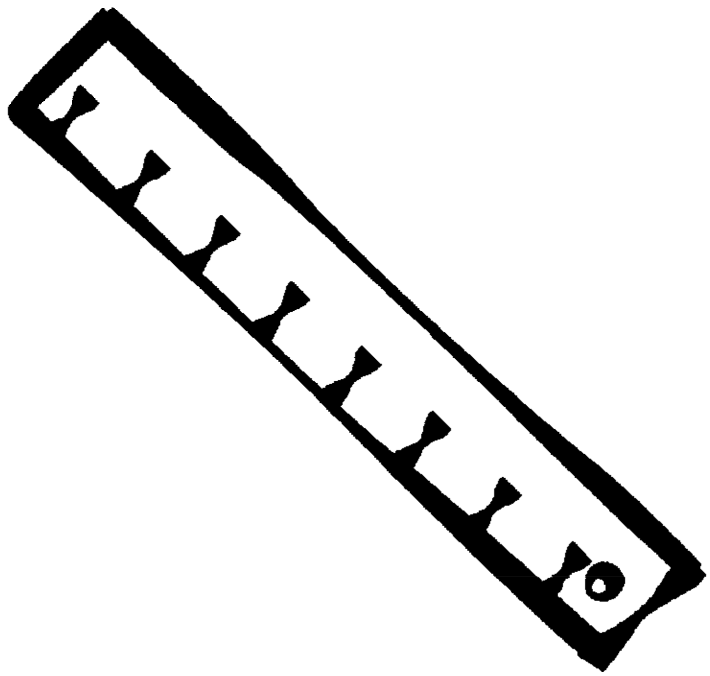
[C]	Communication	**[PS]**	Problem Solving
[CN]	Connections	**[R]**	Reasoning
[ME]	Mental Mathematics and Estimation	**[T]**	Technology
		[V]	Visualization

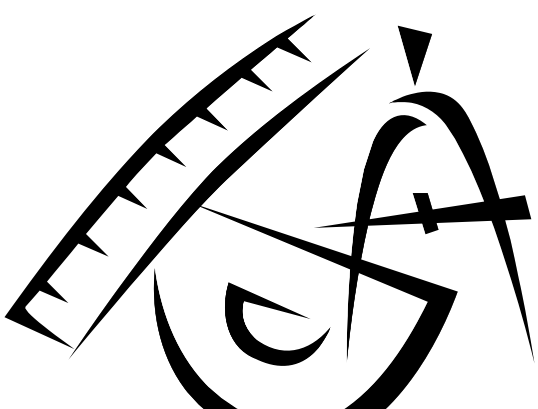
Grade 3 **Strand:** Shape and Space (Measurement)	**General Outcome:** Use direct or indirect measurement to solve problems.
Specific Outcomes *It is expected that students will:*	**Achievement Indicators** *The following set of indicators* ***may*** *be used to determine whether students have met the corresponding specific outcome.*
1. Relate the passage of time to common activities using non-standard and standard units (minutes, hours, days, weeks, months, years). [CN, ME, R]	➢ Select and use a non-standard unit of measure, such as television shows or pendulum swings, to measure the passage of time and explain the choice. ➢ Identify activities that can or cannot be accomplished in minutes, hours, days, months and years. ➢ Provide personal referents for minutes and hours.
2. Relate the number of seconds to a minute, the number of minutes to an hour and the number of days to a month in a problem-solving context. [C, CN, PS, R, V]	➢ Determine the number of days in any given month using a calendar. ➢ Solve a given problem involving the number of minutes in an hour or the number of days in a given month. ➢ Create a calendar that includes days of the week, dates and personal events.
3. Demonstrate an understanding of measuring length (cm, m) by: • selecting and justifying referents for the units cm and m • modelling and describing the relationship between the units cm and m • estimating length using referents • measuring and recording length, width and height. [C, CN, ME, PS, R, V]	➢ Provide a personal referent for one centimetre and explain the choice. ➢ Provide a personal referent for one metre and explain the choice. ➢ Match a given standard unit to a given referent. ➢ Show that 100 centimetres is equivalent to 1 metre by using concrete materials. ➢ Estimate the length of an object using personal referents. ➢ Determine and record the length and width of a given 2-D shape. ➢ Determine and record the length, width or height of a given 3-D object. ➢ Draw a line segment of a given length using a ruler. ➢ Sketch a line segment of a given length without using a ruler.

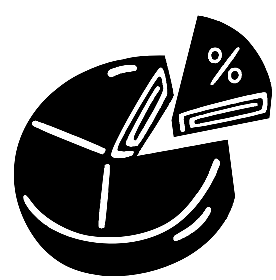
[C] Communication	**[PS]** Problem Solving
[CN] Connections	**[R]** Reasoning
[ME] Mental Mathematics and Estimation	**[T]** Technology
	[V] Visualization

Grade 3 **Strand:** Shape and Space (Measurement) (continued)	**General Outcome:** Use direct or indirect measurement to solve problems.
4. Demonstrate an understanding of measuring mass (g, kg) by: • selecting and justifying referents for the units g and kg • modelling and describing the relationship between the units g and kg • estimating mass using referents • measuring and recording mass. [C, CN, ME, PS, R, V]	➢ Provide a personal referent for one gram and explain the choice. ➢ Provide a personal referent for one kilogram and explain the choice. ➢ Match a given standard unit to a given referent. ➢ Explain the relationship between 1000 grams and 1 kilogram using a model. ➢ Estimate the mass of a given object using personal referents. ➢ Determine and record the mass of a given 3-D object. ➢ Measure, using a scale, and record the mass of given everyday objects using the units g and kg. ➢ Provide examples of 3-D objects that have a mass of approximately 1g, 100g and 1kg. ➢ Determine the mass of two given similar objects with different masses and explain the results. ➢ Determine the mass of an object, change its shape, re-measure its mass and explain the results.
5. • Demonstrate an understanding of perimeter of • regular and irregular shapes by: • estimating perimeter using referents for centimetre or metre • measuring and recording perimeter • (cm, m) • constructing different shapes for a given • perimeter (cm, m) to demonstrate that many • shapes are possible for a perimeter. C, ME, PS, R, V]	➢ Measure and record the perimeter of a given regular shape, and explain the strategy used. ➢ Measure and record the perimeter of a given irregular shape, and explain the strategy used. ➢ Construct a shape for a given perimeter (cm, m). ➢ Construct or draw more than one shape for the same given perimeter. ➢ Estimate the perimeter of a given shape (cm, m) using personal referents.

Research







Helping children develop key ideas in their learning of measurement

Andrea McDonough
Australian Catholic University

The use of measurement is a prominent feature of our lives, often involving estimation (Northcote & McIntosh, 1999; Zevenbergen & Zevenbergen, 2004), but with actions that affect precision and accuracy being important elements of the measurement process (Wilson & Osborne, 1992) especially in occupations such as nursing (Gillies, 2004). The science of measurement is inherently imprecise, due to characteristics of measurement devices and the way people use these devices (Lehrer, 2003; Wilson & Osborne, 1992). Judgment, perception and decision making, such as to the size of the unit, contribute to this imprecision. It is important that children understand such processes and their involvement in these as a part of measurement and that they have a good understanding of a range of key measurement ideas that underpin what they do.

Key ideas of measurement

In learning about measurement during elementary schooling, students learn about several attributes of measurement including length, area, angle and volume. As stated by Wilson and Osborne (1992):

> Although measurement concepts are complex and cause children a variety of difficulties, the basic idea of direct measurement is quite simple. A continuous property such as area, length, or angles must be subdivided into discrete parts so they can be counted. Second, the unit is repeated, dividing the object into several subdivisions with perhaps a fraction of a unit left over. Finally the units are counted to produce a measurement of the object. Indirect measurements such as temperature, rate or density are more complex but still involve counting appropriate units. (p. 91)

Stephan and Clements (2003) warn us that the complex mental accomplishments within measuring are often downplayed in typical measurement teaching. Kamii and Clarke (1997) believe that typical instruction focuses more on measurement as an empirical procedure, such as placing paper clips along a pencil and counting them, rather than a procedure requiring reasoning.

Identification of key or big ideas in measurement can help us consider the reasoning that is required.

In 1992 Wilson and Osborne proposed a set of six foundational ideas of measure, the teaching of which they believed could help children develop their understandings of specific measurement systems and help children transfer concepts, where appropriate, from one measurement system to another. In a more recent paper, Lehrer (2003) wrote of eight components that he saw as "some of the most prominent conceptual foundations" of measure and that he believed form the basis for "a network or web of ideas related to unit" (p. 181). A publication by Lehrer, Jaslow, and Curtis (2003) described seven important ideas specifically in length measurement, divided into two categories: conceptions of unit and conceptions of scale. The big ideas discussed below as listed by Stephan and Clements (2003) also relate specifically to the concept of Length. Kamii and Clarke (1997) remind us that the purpose of measuring length is to compare two things that cannot be compared directly. Informed by the work of Piaget and their own research, they believe that the two mental abilities of transitive reasoning and unit iteration are necessary in

measuring of length. Key measurement ideas proposed by these authors will now be discussed, but with the discussion focusing largely on the concept of length, as this is the first attribute usually encountered by children in their formal learning of measurement at school. The application, or not, of these concepts to different attributes of measure is not discussed in detail but it is acknowledged that other lists might be given. For example, for Area, Stephan and Clements (2003) discuss partitioning, unit iteration, conservation, and structuring an array as four foundational concepts.

The key ideas identified by the different authors are listed in Table 1 and then described in more detail below (in alphabetical order).

Table 1: *Key measurement ideas identified by different authors*

Measurement		**Length**		
Lehrer (2003)	**Wilson & Osborne (1992)**	**Kamii & Clarke (1997)**	**Lehrer et al. (2003)**	**Stephan & Clements (2003)**
Unit-attribute relations Iteration Tiling Identical units Standardisation Proportionality Additivity Origin (zero-point)	Number assignment Comparison Congruence Unit Additivity Iteration	Unit iteration Transitivity	Conceptions of unit: - Iteration - Identical unit - Tiling - Partition - Additivity Conceptions of scale: - Zero-point - Precision	Partitioning Unit iteration Transitivity Conservation Accumulation of distance Relation to number

Additivity: Wilson and Osborne (1992) state that, “measurement of parts can be added to obtain the measurement of the whole” (p. 94). Lehrer (2003) and Lehrer et al. (2003) also identify this as an important measurement idea, noting, for example, that a line segment can be divided into smaller line segments whose sum equal the original length (Lehrer, 2003). Importantly, Lehrer also identifies the key idea that although two paths may begin and end at the same point, their lengths may be different because the sum of parts of one path are greater than the sum of parts for the other path. Related to conservation is the understanding that the length of an object is not affected if it is moved to a new point.

Accumulation of distance: Stephan and Clements (2003) defined accumulation of distance as meaning “the result of iterating a unit signifies, for students, the distance from the beginning of the first iteration to the end of the last” (p. 6).

Comparison: Wilson and Osborne (1992) refer to the importance of understanding that “like properties can be compared to see which is greater”. They state that, “the method of comparing properties varies from measurement system to measurement system” (p. 92).

Congruence: Once again, this key idea is identified only by Wilson and Osborne (1992) who state that "figures can be compared if they coincide when superimposed" (p. 93). They note that while it is reasonable to think about congruence in relation to length and area, it does not apply to systems such as temperature or time as it applies only to geometric figures.

Conservation: Conservation relates to an object retaining its size when moved or sub-divided (Wilson & Rowland, 1993). Stephan and Clements (2003) discuss conservation specifically in relation to length: "conservation of length is the understanding that as an object is moved, its length does not change" (p. 5).

Identical unit: It is stated that units or subdivisions must be identical, as then a count can represent the measure (Lehrer, 2003; Lehrer et al., 2003). When mixed units are used, such as metres and centimetres, each must be named specifically, along with the appropriate number, for example, something may be 2m and 35 cm tall, not 37 tall. Wilson and Osborne (1992 imply the importance of identical units when speaking of the iteration of the unit, that is, iteration of the same (or identical) unit.

Iteration: This foundational idea refers to the subdivision of the whole into congruent parts and translating that unit successively (Lehrer, 2003). Wilson and Osborne (1992) state that, "A unit can be repeated to 'cover' the property being measured. The number of iterations is the number assigned to that measurement" (p. 94). Kamii and Clarke (1997) make it clear that unit iteration is demonstrated when only one unit is used and moved along the length, that is, the one unit is used repeatedly. For example, the length of a small block is seen as part of a whole and then used repeatedly. They believe that unit iteration is constructed out of transitive reasoning, in that, where transitivity involves the comparing whole units, unit iteration involves making a part-whole relationship between two wholes.

Number: Wilson and Osborne (1992, p. 92) state that "for every measurement there is a single number to represent that measurement. The number reports how many units in that measurement". Although Number is not listed as a key idea by Lehrer and his colleagues, the importance of the number appears to be inherent within the concept of unit. Indeed it with units that the number becomes meaningful.

Origin/Zero point: Another important understanding that has been identified is that when using a scale to measure it is important to identify the zero point, and that any point can serve as the zero point or origin on a scale (Lehrer, 2003; Lehrer et al., 2003). For example, as stated by Lehrer (2003,) the distance between 0 and 10 is the same as the distance between 30 and 40. Under the heading of Conceptions of Scale, Lehrer at al. (2003) include Zero-point.

Partition/ing: When speaking of units, Lehrer et al. (2003) add that an important understanding is that units can be partitioned. Stephan and Clements (2003) describe partitioning as "the mental activity of slicing up the length of an object into the same-size units" (p. 4). They note that partitioning a unit is non-trivial for students, as they have to mentally see that the unit can be cut up before they physically do this. Stephan and Clements speak also of students eventually coming to understand that length is continuous and can be further partitioned.

Precision: As discussed above, all measurement is approximate. Lehrer et al. (2003) point out that a key idea for children to come to understand is that the choice of unit determines the level of precision. This understanding of precision is categorised under Conceptions of Scale.

Proportionality: Again, only Lehrer (2003) identifies this as a key idea within measurement. This understanding is that different-sized units can be used to measure the same thing. Thus different quantities can represent the same measure, these quantities being inversely proportional to the size of the units used.

Relation to number: Stephan and Clements (2003) state that an important understanding is that "Measurement is related to number in that measuring is simply a case of counting. However, measurement is conceptually more advanced since students must reorganise their understanding of the very objects they're counting (discrete versus continuous units)" (p. 7).

Standardisation: Only Lehrer (2003) identifies Standardisation as a key idea within measurement. He refers to the use of standard units facilitating communication of measures.

Tiling: Tiling refers to the idea that the units must fill the space, that gaps must not be left between the units (Lehrer, 2003; Lehrer et al., 2003) and that there should be no overlaps (Wilson & Osborne, 1992). As stated by Lehrer (2003, p. 181), "Tiling (space-filling) is implied by subdivision of lengths, areas, volumes, and angles, but this implication is not apparent to all children".

Transitivity: Transitive reasoning applies when comparing two items where direct comparison is not possible (Kamii & Clarke, 1997; Stephan & Clements, 2003). Use of a third item allows comparison, and is dependent on understanding of the following relationships:
If Length A > Length B, and Length B > Length C, then Length A > Length C
If Length A < Length B, and Length B < Length C, then Length A < Length C
If Length A = Length B, and Length B = Length C, then Length A = Length C
Transitivity reasoning involves deducing a relationship from two or more relationships of equality of inequality (Kamii & Clarke, 1997).

Unit/Unit-attribute relations: A key idea identified by a range of researchers is the understanding of unit. Wilson and Osborne (1992) and Lehrer (2003) identified the importance of the unit in terms of what is to be measured, that is, the unit needs to be compatible with the property or attribute to be measured. Lehrer (2003) referred to this as Unit-attribute relations.
Lehrer et al. (2003) included the concept of unit as a category within which they placed the ideas of Iteration, Identical unit, Tiling, Partition, and Additivity.

While there is some variation in the important measurement ideas identified by researchers, these can be informative for teachers. For example, Lehrer (2003) discusses his eight components of measurement in relation to the measurement concepts of length, area, volume, and angle. Lehrer (2003) believes that "studies conducted in the last two decades suggest that children's developing sense of measurement is marked by gradual coordination and consolidation of these components" (p. 182). He suggests also that understanding of the eight components can be extended from one attribute to another. Wilson and Osborne (1992) also believe that developing

understanding of the six foundation ideas identified by them can save time if children are encouraged to look for transfer from one measurement system to another. However, they also note that the six foundational ideas that they have discussed do not all apply to all measurement attributes or systems.

Background

Much research on the learning of measurement is influenced by the work of Piaget and his colleagues. Piaget, in identifying stages of development in coming to understand measurement concepts such as conservation, the idea of a unit, transitivity, and iteration (Carpenter, 1976; Wilson & Rowland, 1993), focused on the development of cognitive abilities within the individual.

Piaget believed that conservation is a necessary part of understanding the measurement process. As stated above, conservation relates to an object retaining its size when moved or sub-divided. A child who is not conserving makes one-dimensional perceptual judgements. For example, when comparing lengths the child makes visual comparisons only, pays attention to end points and does not take account of undulations (Carpenter, 1976). Children come to understand that, when measuring, a unit is taken from the whole and transposed onto the whole through subdivision and change of position. Within this process the size of the unit remains the same (conservation) and the unit is used iteratively, that is, it is seen as part of the whole and is used repeatedly. These are referred to as logical-mathematical invariants of measurement (Nunes, Light, & Mason, 1991).

Piaget proposed also the need for an understanding of transitivity for a child to measure in an operational manner (Carpenter, 1976; Kamii & Clarke, 1997; Wilson & Rowland, 1993) and he proposed age related stages at which children develop understandings of measure (Carpenter, 1976).

There have been a number of studies that confirm or challenge Piaget's findings (Carpenter, 1976; Kamii & Clarke, 1997; Lehrer, 2003; Stephan & Clements, 2003). For example, Carpenter reports that studies of conservation tended to show a common pattern generally confirming the work of Piaget but that the sequence of development of transitivity and conservation, seen as synchronous by Piaget and his colleagues, was not clear within research that has followed.

Further, research on length and area suggests also that these concepts do not necessarily develop simultaneously as reported by Piaget. For example, Nunes, Light, and Mason (1993) found differences in children's abilities to calculate length and area using conventional and non-conventional tools. Kamii and Clarke (1997) confirmed part of Piaget's work but suggest a difference in the age when students construct unit iteration. Wilson and Rowland (1993) refer to studies that found children achieving measurement concepts at different ages from those identified by Piaget and his colleagues. From his review of measurement research, Carpenter (1976) concluded that "it is all but impossible to establish reliable age norms for the emergence of given operations. There are also some serious problems in identifying the sequence of acquisition of different logical operations" (p. 52). Carpenter (1976) concluded also that

> there has been an exaggerated emphasis on internalised logical-mathematical structures ... The research on measurement suggests it is not the existence of internal logical-

> mathematical structures that limits performance. Children possess such structures long before they can apply them. No individual task or group of tasks can conclusively demonstrate the existence or otherwise of a give[n] operation. (p. 69)

It appears also that the Piagetian stages do not lead to clear teaching and assessment guidelines (see Carpenter, 1976; Kamii & Clarke, 1997). It is clear that the development of children's understandings of measurement is complex and that children may possess logical-mathematical structures before they can be demonstrated. Lehrer (2003) reports that studies generally do not support Piaget's belief that the incomplete development of logical reasoning of relations such as conservation and transitivity constrain children's ideas about measure. He recommends that rather than delaying instruction until these have been developed, thinking of measure as a network of key ideas that can be developed through activity and focused reflection can be useful to the teacher. That is, the identification and understanding of key or foundational ideas of measurement can provide a developmental context in which to explore measure.

However, Piaget's analysis that "suggested that conceptions of spatial measure were not unitary but instead consisted of a web of related constructs leading to eventual construction and coordination of standard units" (Lehrer, 2003, p. 180), appears relevant today. The identification of key measurement ideas suggests, for example, that it is important to distinguish, as did Piaget, "between activity, such as using a ruler, and reflective abstraction on activity, such as understanding the role played by the identical units in the ruler" (Lehrer, 2003, p. 180).

Implications for teachers

Curriculum documents have traditionally included elements such as awareness of measurement attributes, comparison, use of non-standard units and use of standard units. These may suggest a simplicity to measurement. But it is important that teachers are also aware that a web of ideas underpins the understanding of these elements as the apparent simplicity may lead teachers to underestimate the complex mental accomplishments involved (Stephan & Clements, 2003).

Indeed, the learning of measurement is complex and requires focused activities that include development of appropriate language and elements of reflection that help children draw out the key ideas from activities. A focus on measurement skills, that is, on procedural competence is not enough. An understanding of concepts must also be associated with this. It is useful to refer to a study of the teaching of length in the first year of school where Sullivan and McDonough (2002) found that "the most effective teachers seemed able to articulate focused, developmentally appropriate and engaging activities for their students, and engage them actively in interrogating those experiences" (p. 255). Choosing rich experiences, being clear on the purpose of those experiences, and probing and challenging children's thinking to help them develop measurement understandings were characteristics of these teachers (McDonough, 2002). Intensive case studies of six highly effective early years teachers within the Early Numeracy Research project (ENRP), a three year study conducted with teachers and children in the first three years of school in 72 schools, gave further insights into of the practices effective teachers. Thorough and careful analysis resulted in a list of 25 features common to the mathematics teaching of these teachers (McDonough & Clarke, 2003). Although the common themes are not listed here in their original and complete wording, it is appropriate to summarise some of those

themes by saying that the outstanding teachers

- had a clear mathematical focus;
- used open-ended tasks;
- asked high level questions;
- challenged without threatening; and
- had high but realistic mathematical expectations.

It is contended that these characteristics of highly effective teachers can apply at grade levels beyond the early years (McDonough & Clarke, 2003) and can relate equally to Measurement as to any other part of the mathematics curriculum.

Effective teaching of measurement in action

The following discussion considers aspects such as being clear on the focus of a lesson, engaging the children in rich activities, using open-ended tasks to encourage children to think hard and modify their thinking, probing children's understandings, and helping children reflect on their activity so as to draw out important measurement ideas.

Among other items, each of the ENRP effective teachers of Length in the first year of school was asked to describe an example of an activity they used in their teaching. As stated above, a particular feature of the teachers was that they seemed able to describe rich experiences for the students and the purpose of those experiences. For example, one teacher of children from largely non-English speaking backgrounds, described a series of lessons following the reading of the storybook *The Long Red Scarf* (Hilton, 1987):

> Well my favourite one … "The Long Red Scarf" and I based the series of lessons on that covering the different [length understandings and skills] … I had a whole lot of teddies that the children made scarves for and we compared lengths and then we actually taught them how to measure using blocks and bears and things and we measured our scarves and … language because a lot of our children do not have the language so even simple things like longer and shorter …
>
> I started with reading the book and we talked about scarves, then I brought in scarves and we put them on the floor in the middle of the big circle and I spread them out haphazardly and I said well "which scarf here is the longest?" and the children said "have a guess at anything sort of thing" and I have got a very bright boy who said "no you can't do it like that, you have to line them up" and he lined them up and then someone else said "no but you've got to match them at the end" so they matched them at the end … so we got lots of language.
>
> The second lesson … we actually said we were going to make scarves so I gave them paper. … they had to make a scarf long enough to go around their teddy … they proceeded to make their scarves and some of them even decorated them and then we actually compared lengths again so that was all one lesson, they came back and they put their scarves down and we talked about who had the longest and some of them had very long scarves because they had bigger teddies and some had short scarves.

It seems that this teacher had a clear vision of the experiences that were needed, was able to engage the students through the use of a picture story book and related activities, and was not deterred from such a rich experience by the unfamiliarity of some of her students with the language demands. Other effective teachers gave similarly rich examples.

Another common theme was that these teachers were prepared to probe the thinking and understanding of the children. For example, in response to the same prompt another teacher said:

> I always try and make sure that there's a sharing of findings at the end of each session …and I always ask the kids "how did you obtain such a result?" or "how did you get your answer?". So there's that constant reflection … "if you measured your foot and you found out that it was 22" … also I try and challenge the kids by asking them "if we've all measured our feet and we've all measured the length of a basketball court and we've all got a different response, why is this?" so I'm actually getting them to think a little bit beyond just obtaining a result.

In other words, the teachers seemed to be aware of characteristics of rich experiences and how to use those experiences to extend the students' thinking. In speaking of getting children to think beyond the result, the teacher demonstrated an example of having high but realistic mathematical expectations and of challenging the children. Effective case study teachers were observed to challenge without providing threat for the learners.

The above quote illustrates also that with the use of open-ended tasks it is possible to have a meaningful discussion following the activity. In this case children discussed different answers from measuring the basketball court, thus the teacher could help them draw out the importance of measurement principles such as the need to use a common unit, agreement on starting and finishing points, and the importance of measuring in a straight line and not leaving gaps. Such open-ended tasks can be developed for the different mathematical domains (e.g., see Sullivan & Lilburn, 2004).

Other teachers in the ENRP first year of school classes interviewed regarding Length spoke also of the importance of reflection, and of focusing on "try[ing] not to do as much teaching at the beginning of the lesson [but] doing the teaching at the end of the lesson" and "allow[ing] more time at the end to draw things together and to discuss with children". One teacher stated:

> I think that reflection time is so important, I actually have it planned out more or less what I am going to quite often ask the children or what I hope I will achieve. I think it's too easy sometimes just to come up with something a bit airy fairy, sometimes I like to be quite specific.

This quote suggests that the teacher had identified a clear mathematical focus for her Length lessons. To plan the reflection she needed to be clear on exactly what were the mathematical understandings she was hoping for the children to develop.

Kamii and Clarke (1997) also wrote of the value of open-ended tasks suggesting that, for example, rather than asking how may (centimetres, grams …) something is, it can be more motivational for children to measure two things that can not be compared directly. They give the example of children in their first year of school working out how to compare two rugs that

cannot be compared directly. A further suggestion is to ask children exactly how much paper they will need to bring from another room to cover the bulletin board. The authors indicate that this might be answered, for example, with use of a string (transitive reasoning), with, for example, use of a pencil (transitive reasoning and unit iteration) or, for example with rulers, or a meter stick. Another question they suggest is to ask whether a doorway is wide enough for particular table to fit through. Kamii and Clarke believe that these questions require indirect comparison, and encourage children to struggle with problems and debate among themselves. They believe that teachers should pose tasks that help children to think hard and modify their thinking, rather than simply teach empirical procedures such as counting non-standard units.

In relation to teaching length, McClain, Cobb, Gravemeijer, and Estes (1999) recount episodes in a first grade classroom where children made *Smurf Bars* made from 10 Unifix cubes (representing 10 cans of Smurf food used by Smurfs for measuring) and then measured longer objects by iterating the bar and counting by tens. After much experience with the *Smurf Bar* children were challenged to make a new measurement tool and they created a paper strip the same length that they called a *ten-strip*. Following children's experiences with iteration of the *ten-strip*, the teacher created a *measurement strip* 100 "cans" long. It is reported that in these activities children's reasoning was the main focus, more so than that all students measured correctly. Indeed when the class reflected, the teacher was reported to call on a range of students including students who had reasoned differently. This gave the opportunity for students to listen to each other and to think about what each person had done, thus requiring them to reason further. It is claimed that it also communicated to the students that it was the solution process that was valued. It is noted that the teacher did highlight solution strategies that fitted with her agenda.

Lehrer et al. (2003) stress that if such investigations are not undertaken in an early grade, they should not be skipped over, but illustrate that they can be appropriate for children in grade 5 with the making of *footstrips* and further development such as constructing more finely articulated subdivisions of length. Lehrer et al. state that research indicates that development of conceptual understandings of measurement does not happen spontaneously with age, but requires use of investigations focused on developing students' thinking. Indeed, as students have more measurement experiences and explore the range of measurement attributes, challenges remain in focusing on, and further developing, key understandings.

Measurement poses challenges for teachers and children

The teaching of measurement poses challenges for teachers. Children's needs are identified, appropriate activities with a clear mathematical focus are selected, materials are made ready, and the lesson is taught. Because of the need to develop key measurement understandings as well as the skills of measurement, the teacher plays a major role in facilitating children's learning. For example, during a measurement lesson the teacher asks questions and helps children to notice key ideas of measurement. These actions are informed by teacher knowledge of how children learn, of their own children's understandings and of appropriate pedagogy (e.g., Shulman, 1987). As suggested above, they are informed also by knowledge of key ideas of measurement and of an understanding that although measurement may appear simple, it involves complex mental activity. As stated by Lehrer et al. (2003), our goal is to "help children develop a theory of

measure, as well as practical knowledge of tools such as rulers [so that children will be able to] invent or adapt their ideas to new situations" (p. 103). It appears that there exists "[n]o clear cut 'best' sequence of instruction … in any domain of measure" (Lehrer 2003, p. 190), but research does suggest that focusing on big ideas of measurement can help children develop conceptual understandings as well as procedural knowledge.

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